Colocally connected, Non-cut, Non-block and Shore sets in Hyperspaces and Symmetric Products

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Colocally connected, Non-cut, Non-block and shore sets in Hyperspaces and Symmetric Products

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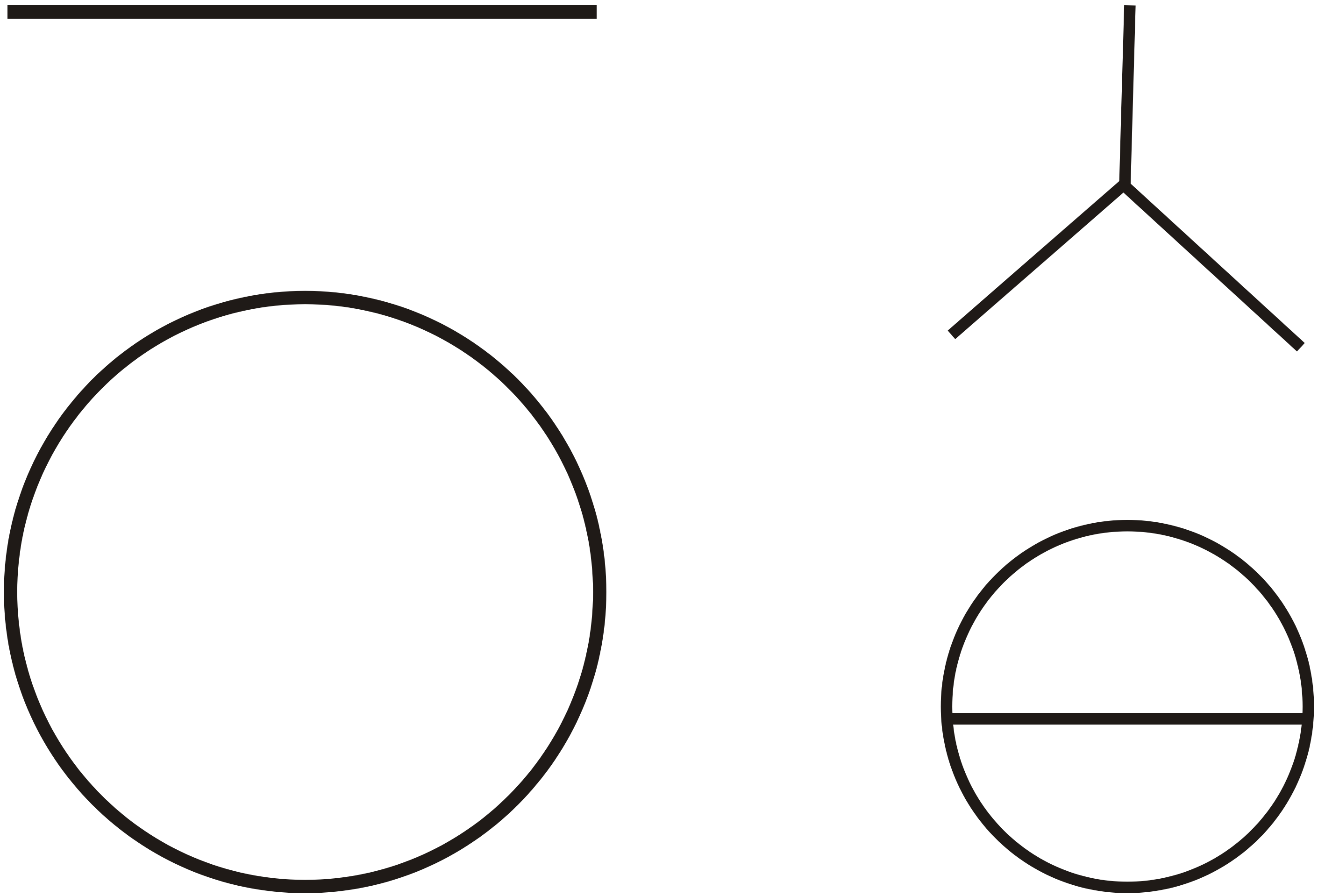
- 3 Main Result in the Hyperspace Cn(X)
- 4. Main Results in Symmetric Products.
- 5. Special Cases in Symmetric Products.
- 6. Counter examples in Symmetric Products.

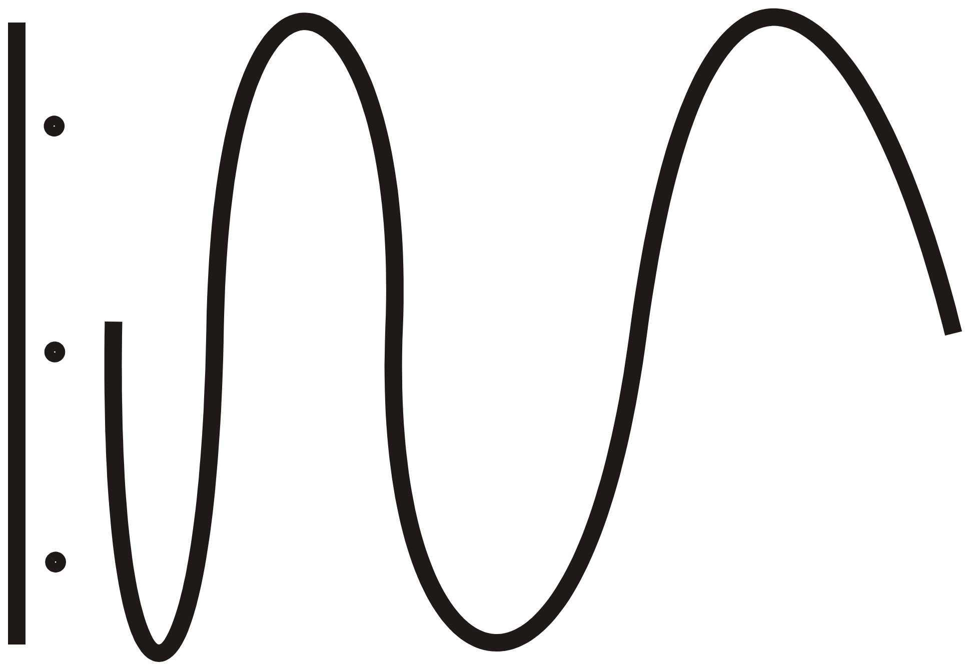
1. Definitions

Definitions

- A ***CONTINUUM*** is a compact connected metric space.

Defintinions-Examples





Sen$(1/x)$-continuum

1.1. Hypersapaces

HYPERSPACES

- Given a continuum X, we define the following hyperspaces:

- $2^X = \{ A \subset X : A \neq \emptyset \text{ and } A \text{ is closed} \}$

The Hyperspaces C(X) and Cn(X)

- $C(X) = \{ A \in 2^X : A$ is connected $\}$
- $C_n(X) = \{ A \in 2^X : A$ has at most n components $\}$

The Symmetric product $F_n(X)$

- $F_n(X) = \{ A \in 2^X : A \text{ has at most } n \text{ points} \}$

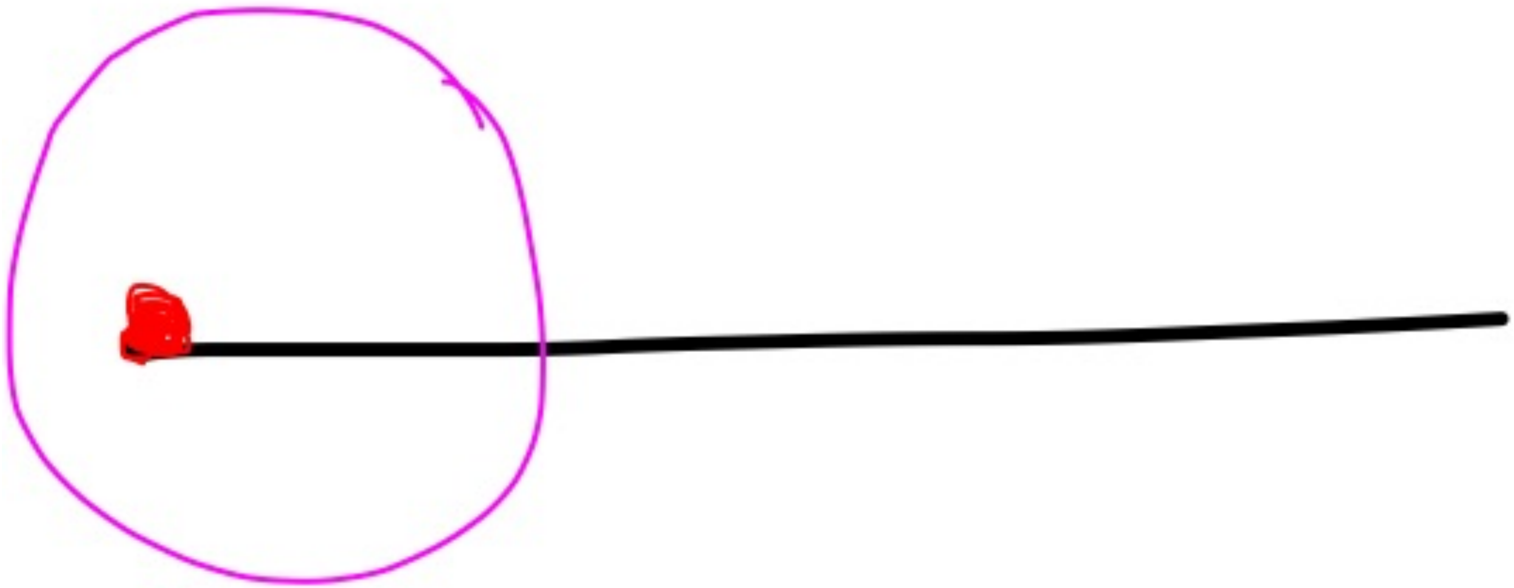
THE HAUSDORFF METRIC IN 2^X

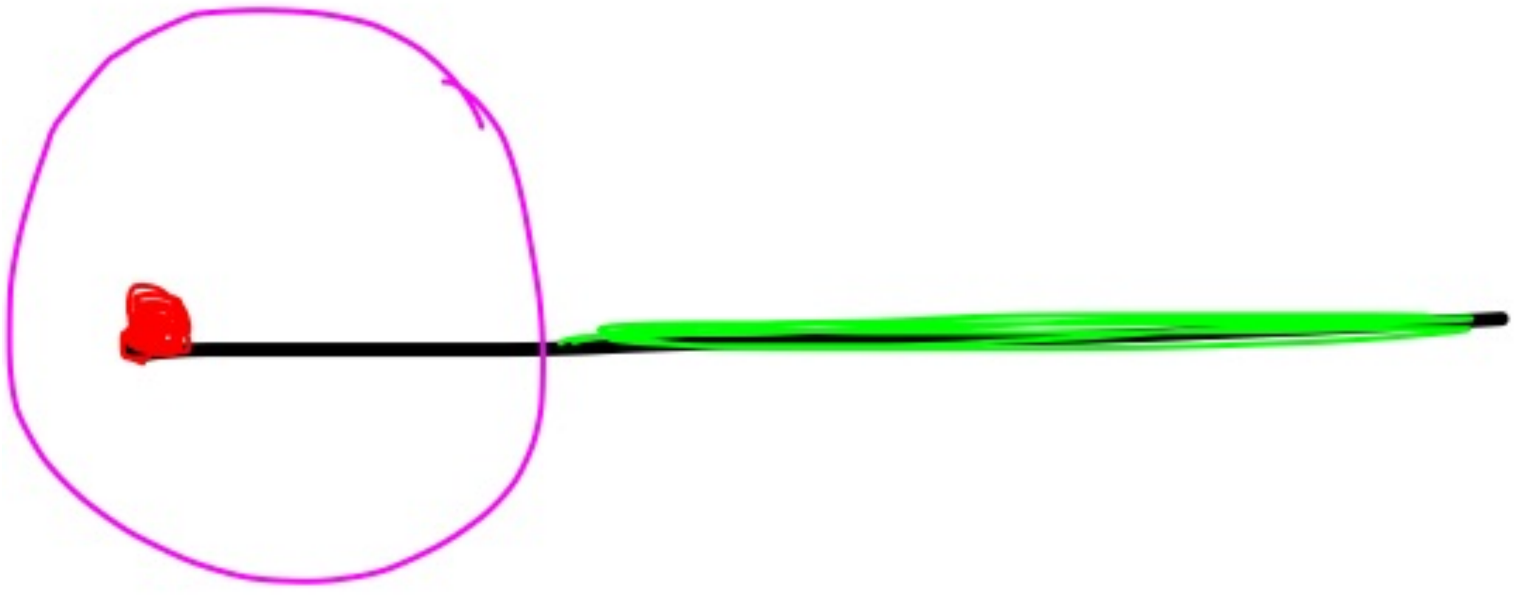
- We endow 2^X with the Hausdorff metric H.
- Since $F_n(X)$, $C(X)$ and $C_n(X)$ are subspaces of 2^X, we endow them with the Hausdorff metric H, as well.

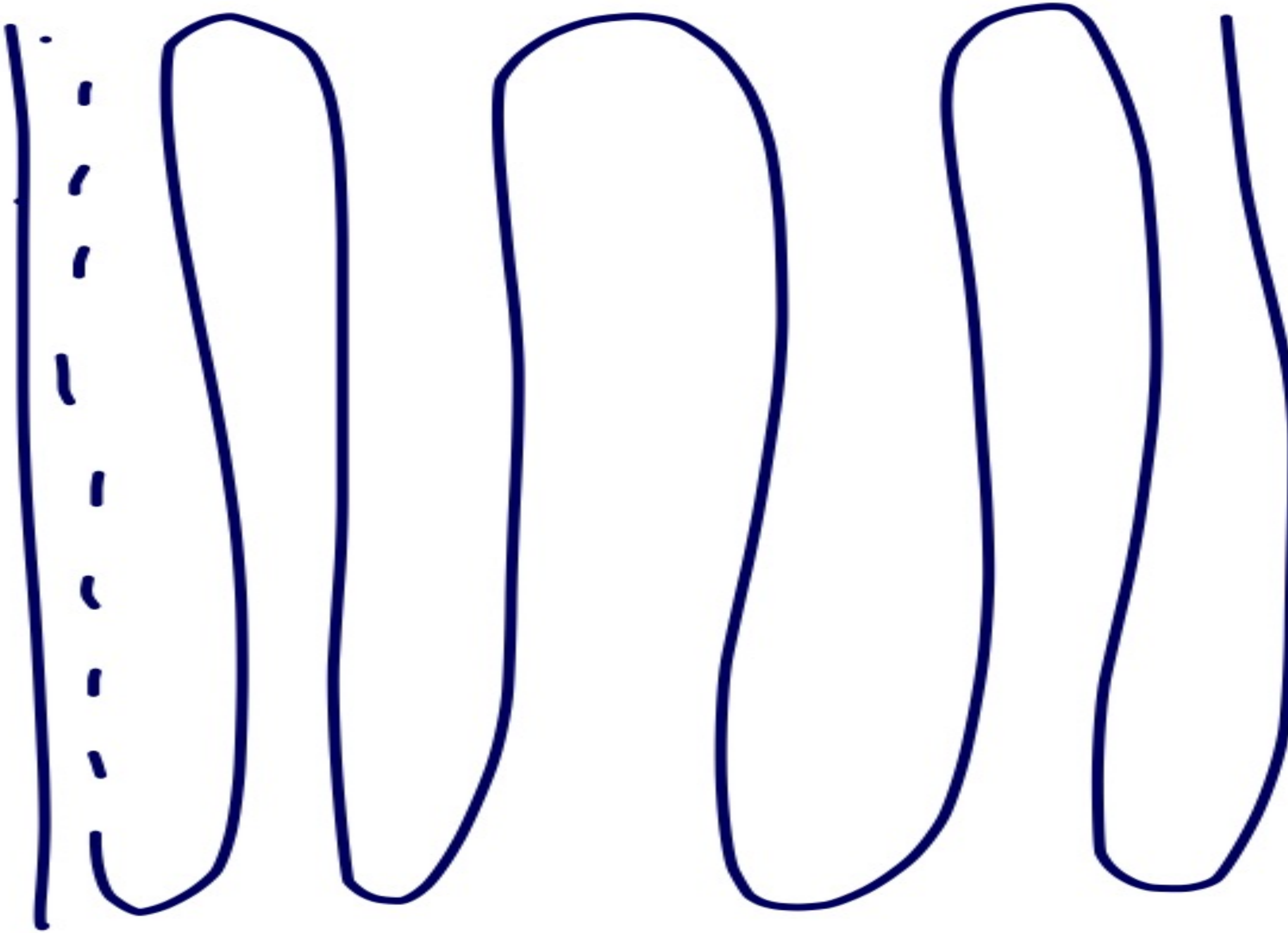
1.3 Colocal connectedness

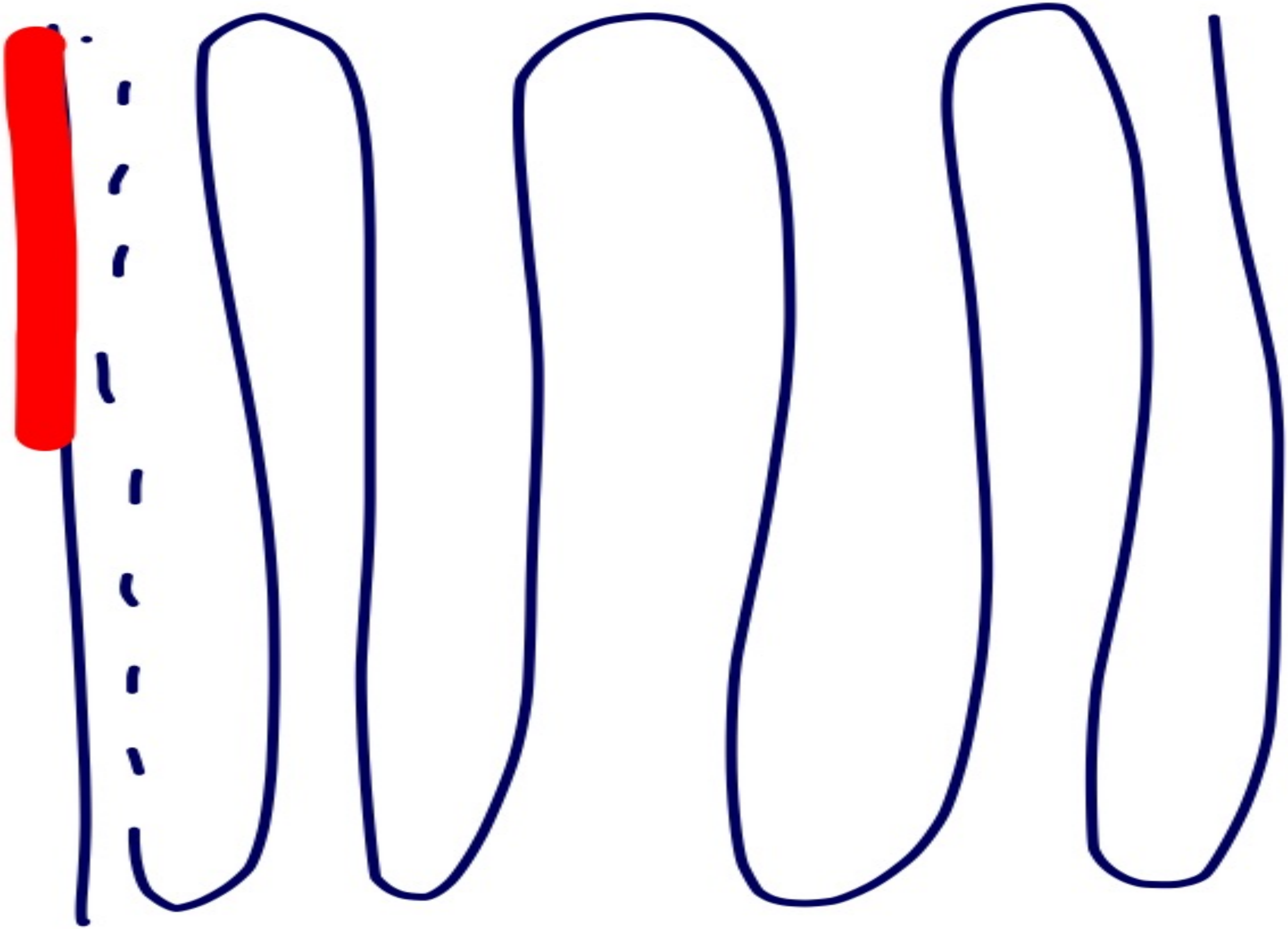
Colocal connectedness

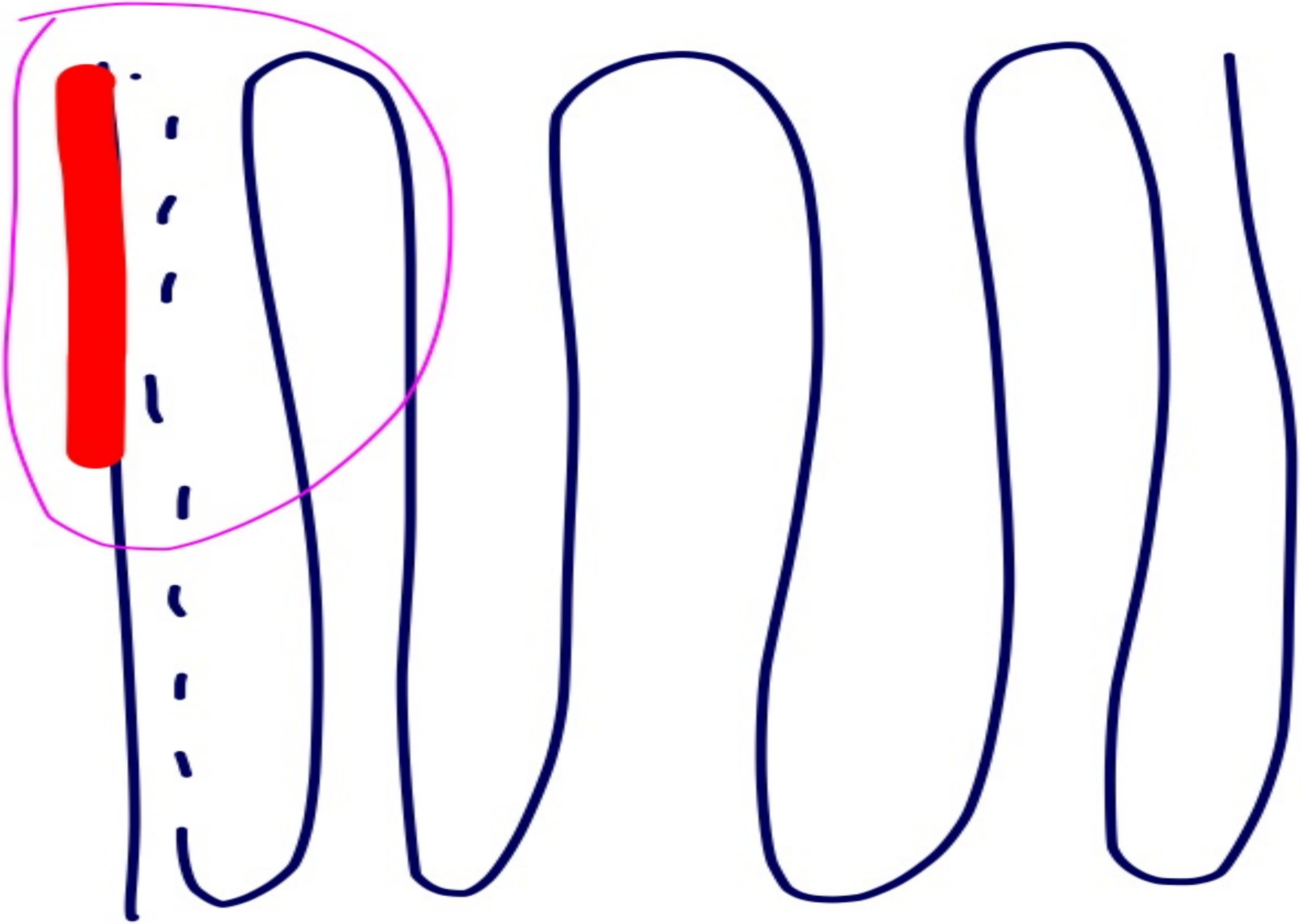
- Let X be a continuum and A a subcontinuum of X with $\text{int}(A) = \emptyset$.
- We say that A is a continuum of **colocal connectedness** in X provided that for each open subset U of X with $A \subset U$ there exists an open subset V of X such that $A \subset V \subset U$ and $X \setminus V$ is connected.

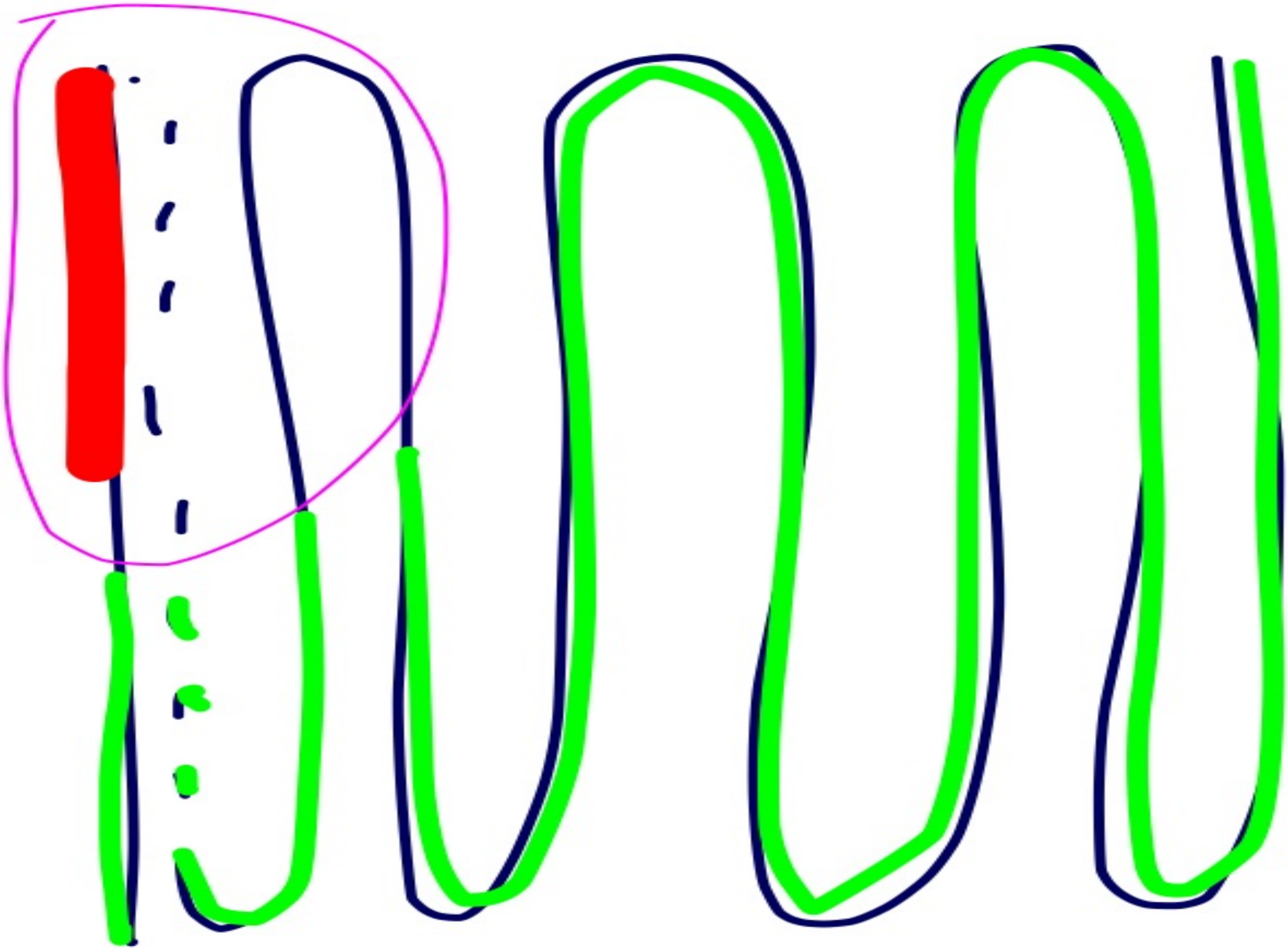


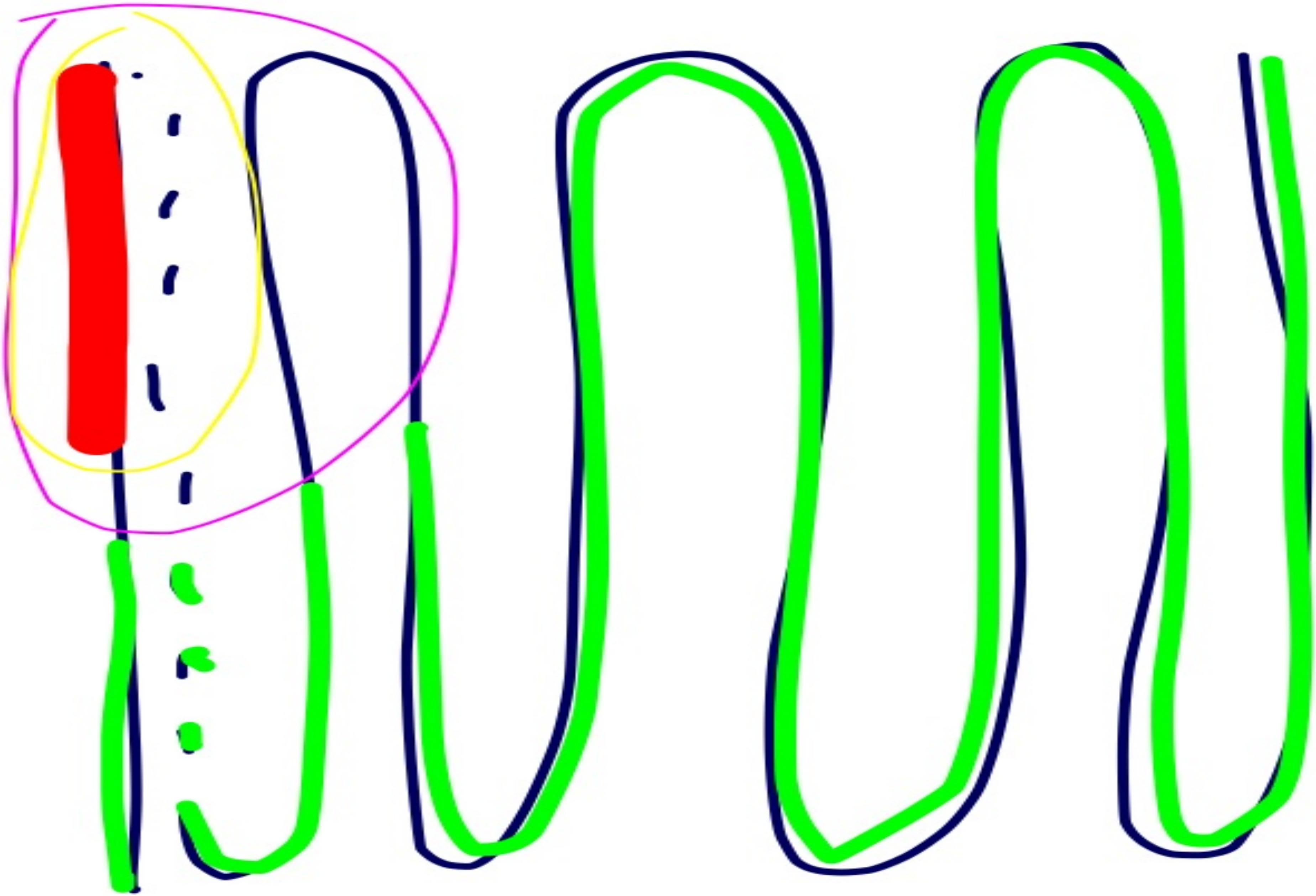


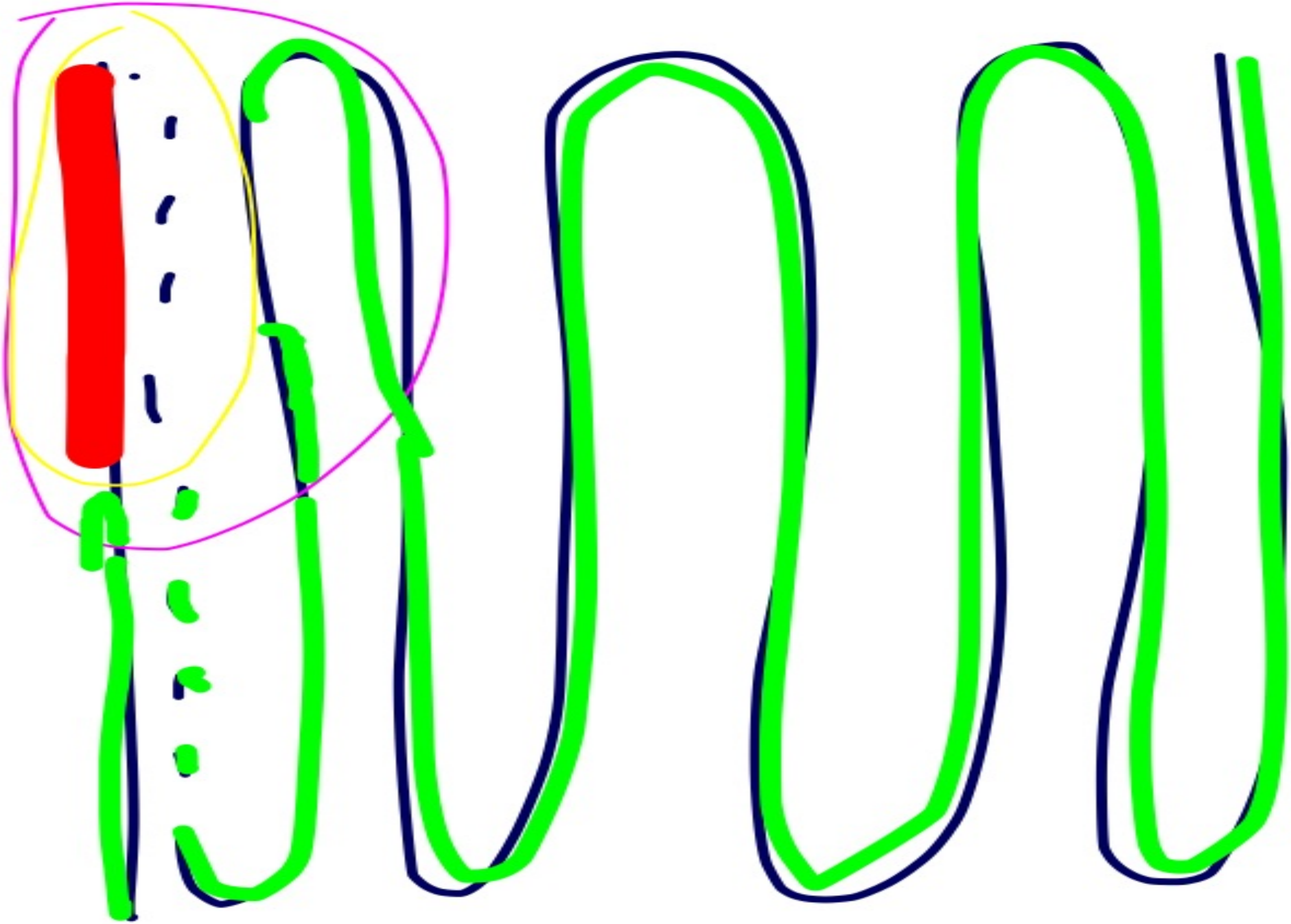


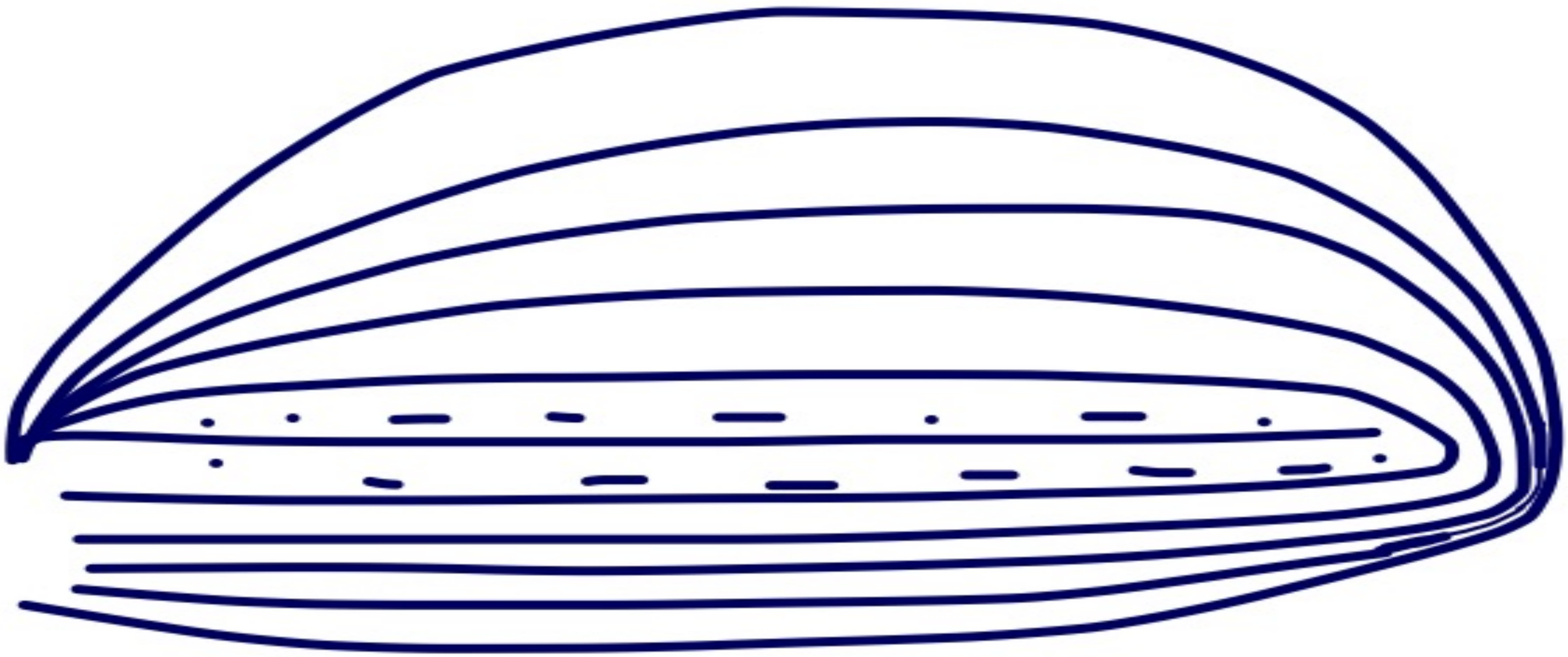


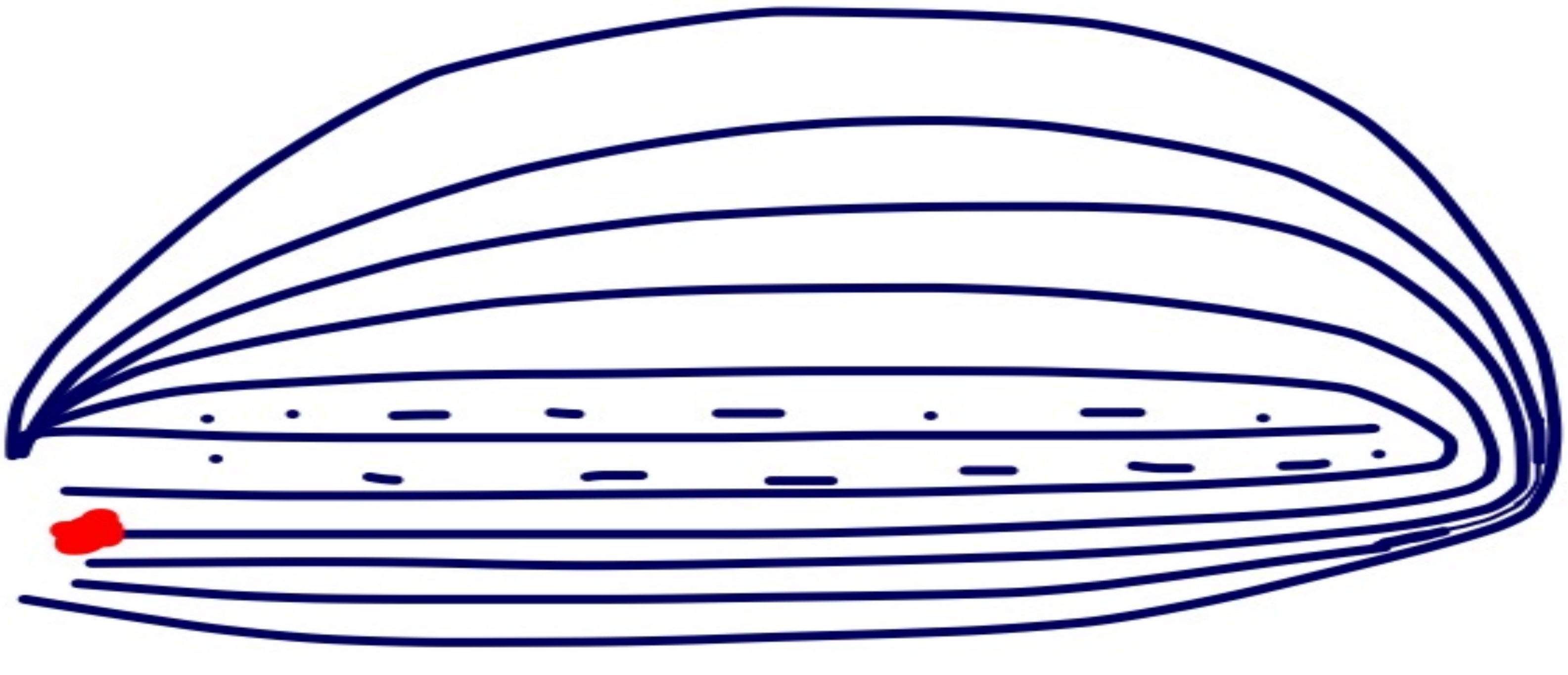


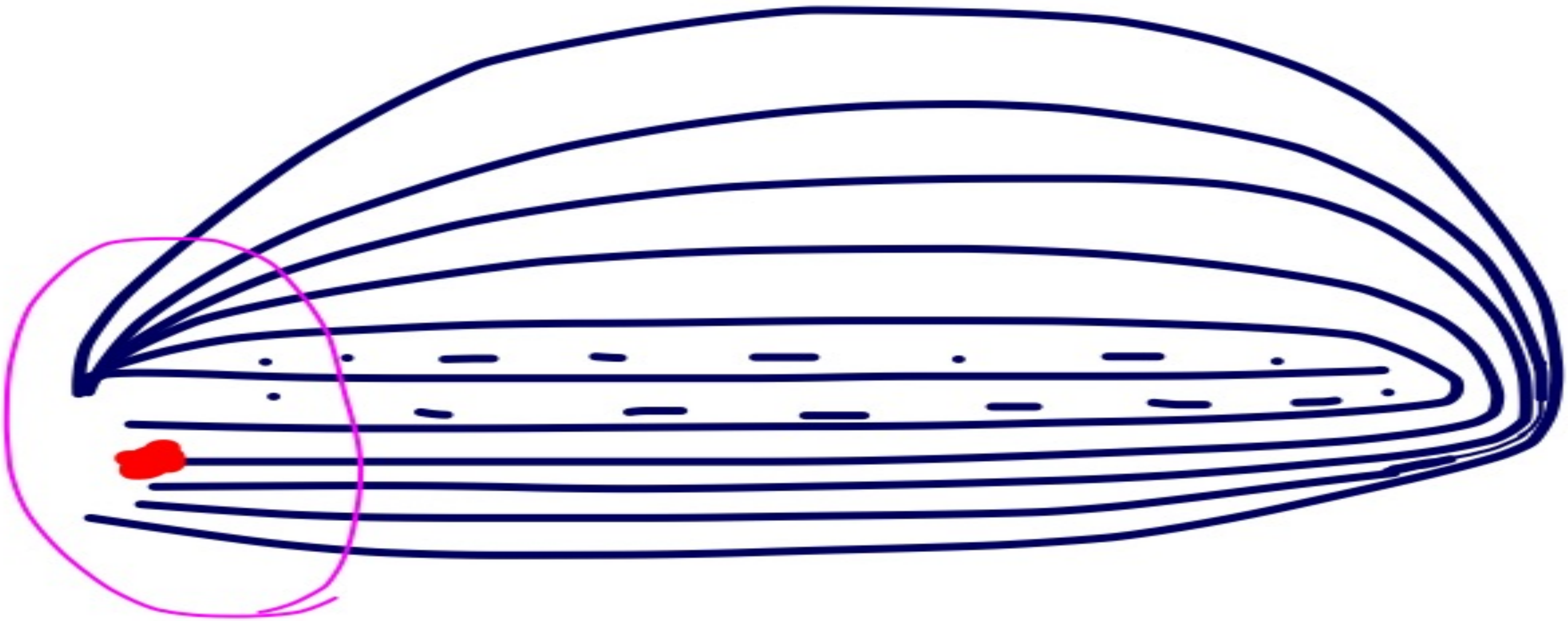


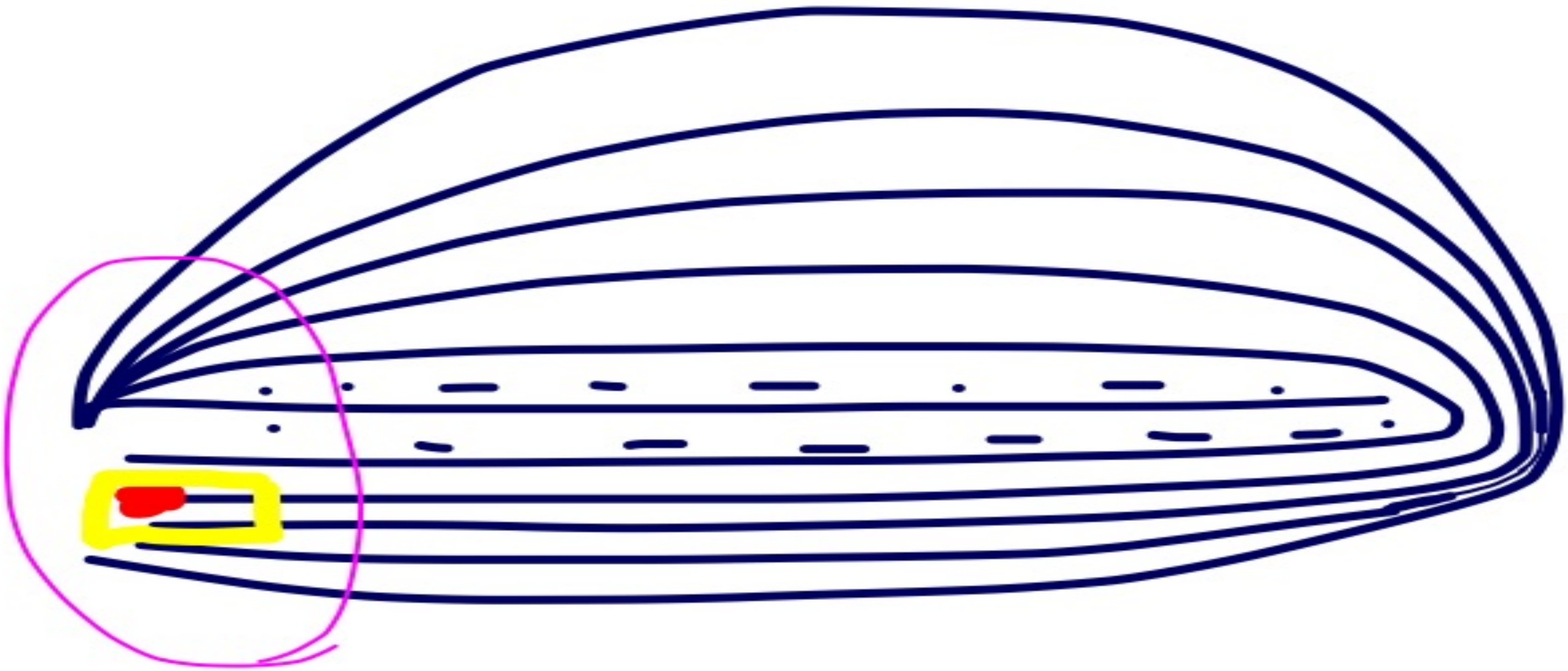


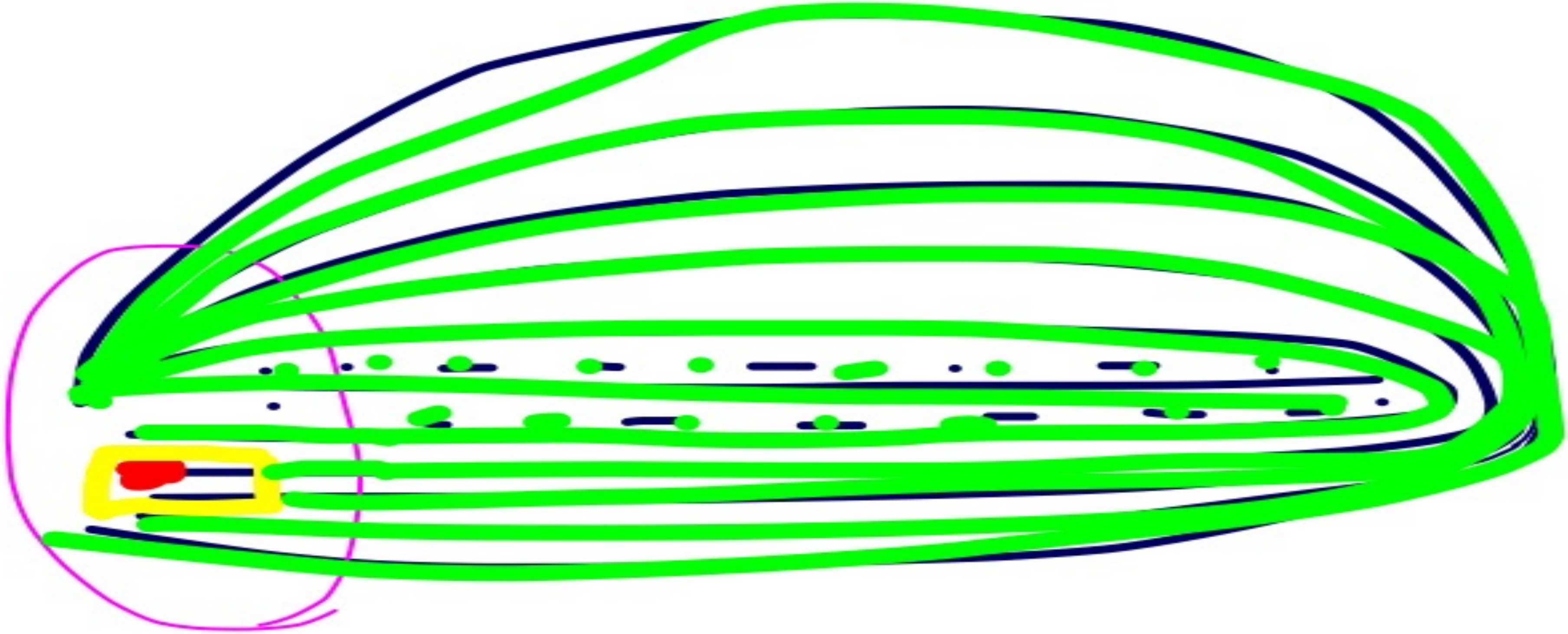


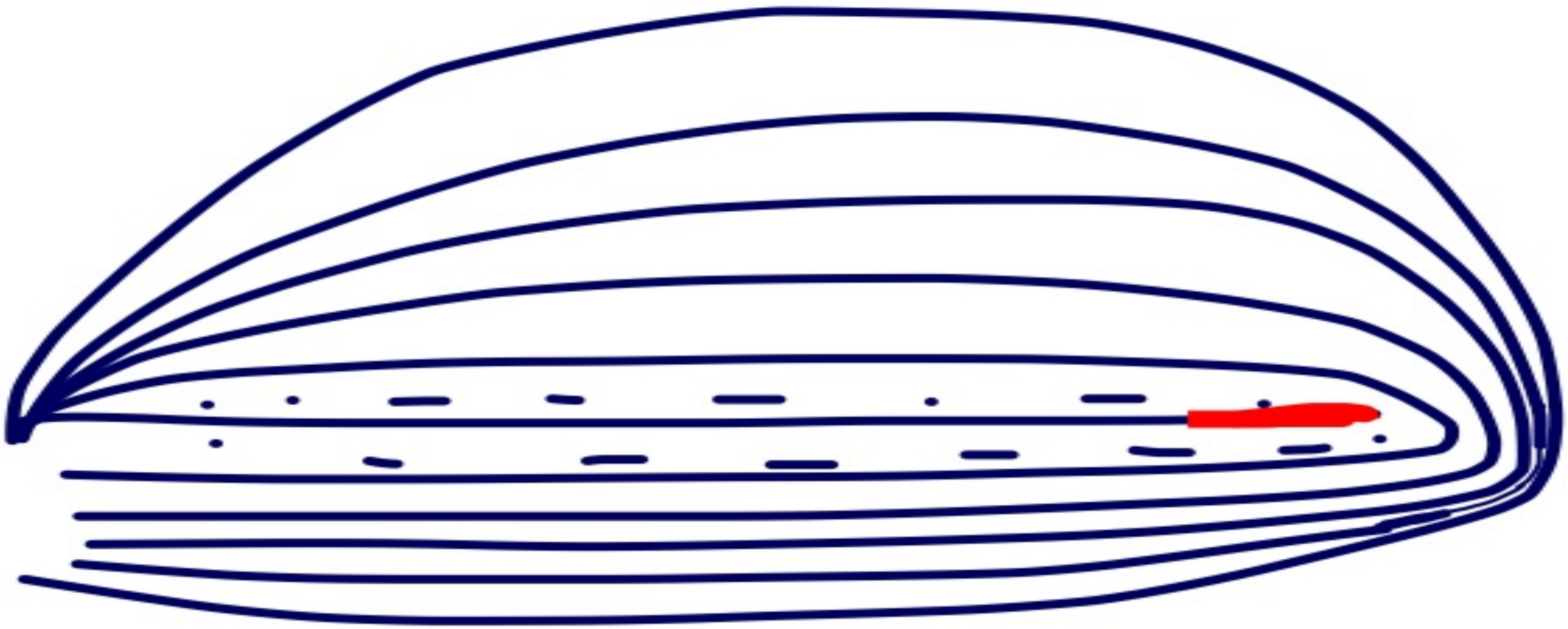


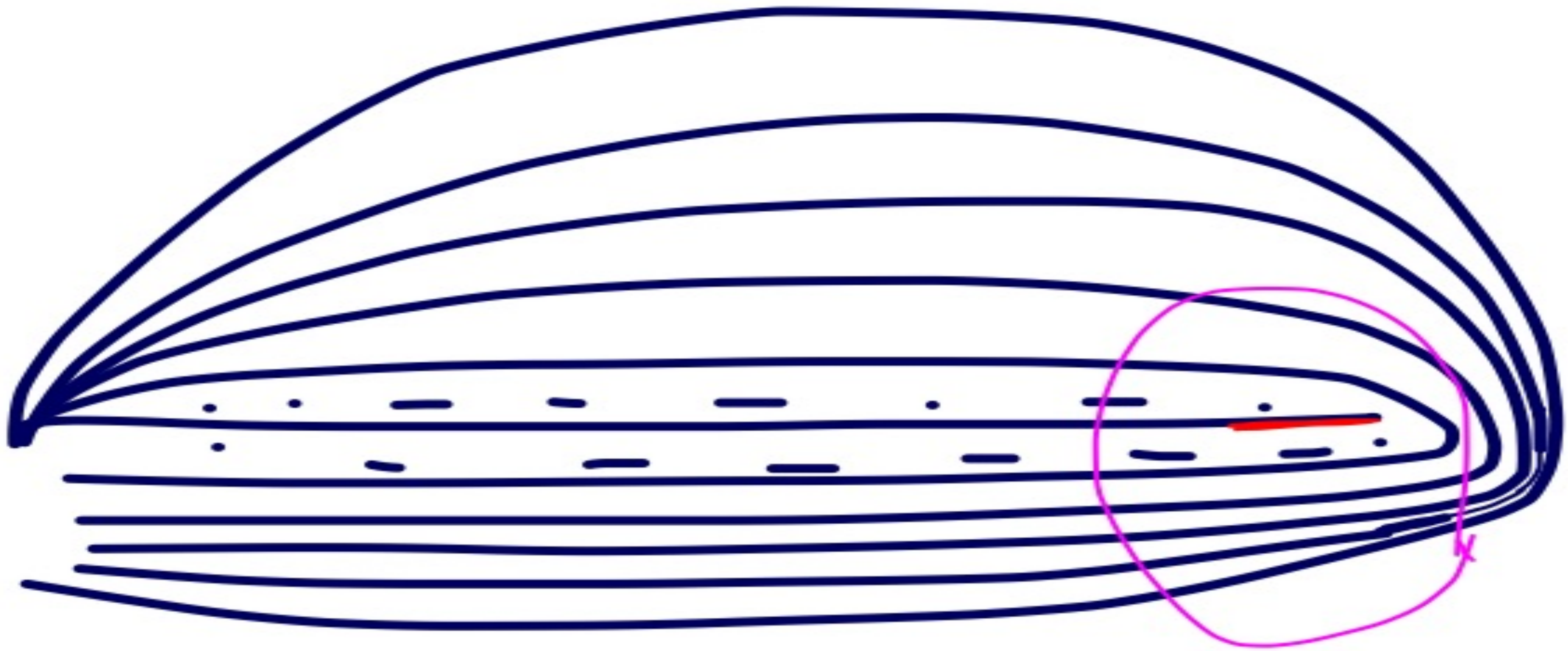


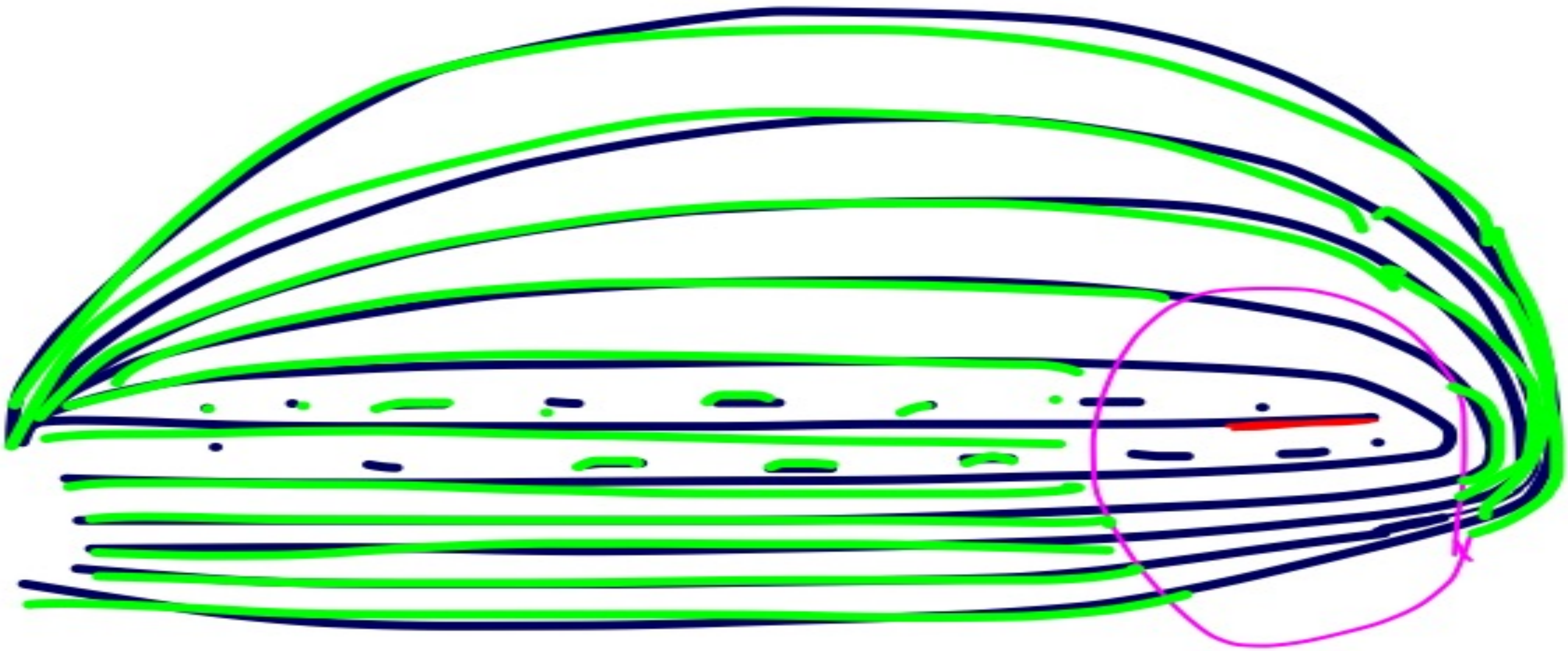












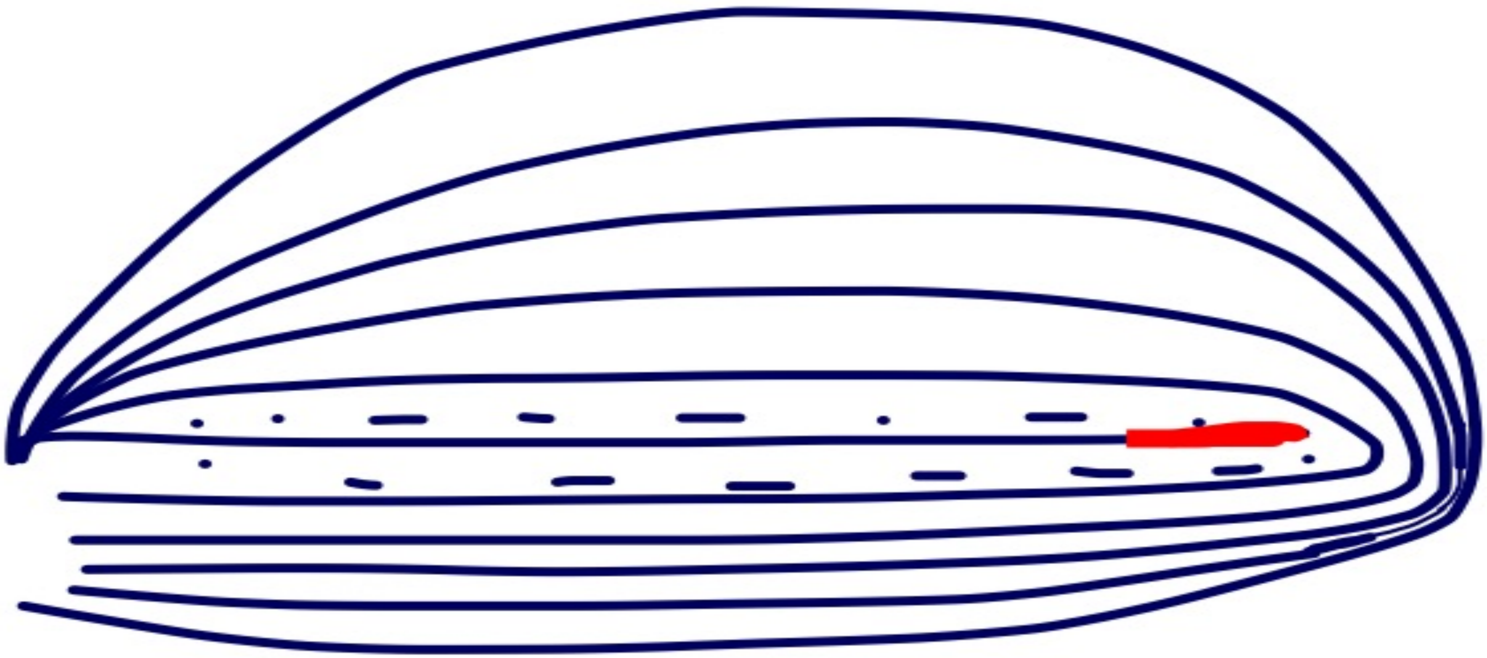
1.4 Not a weak cut continuum

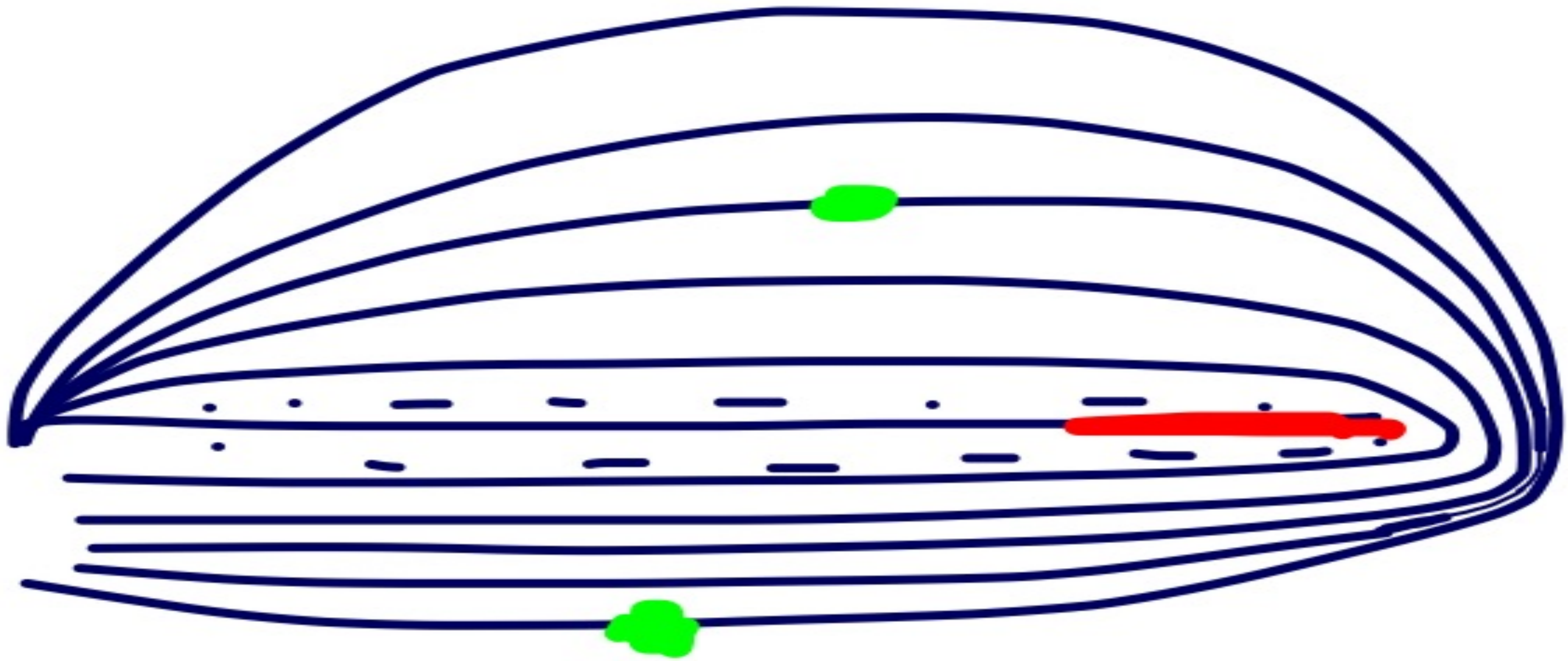
Not a weak cut

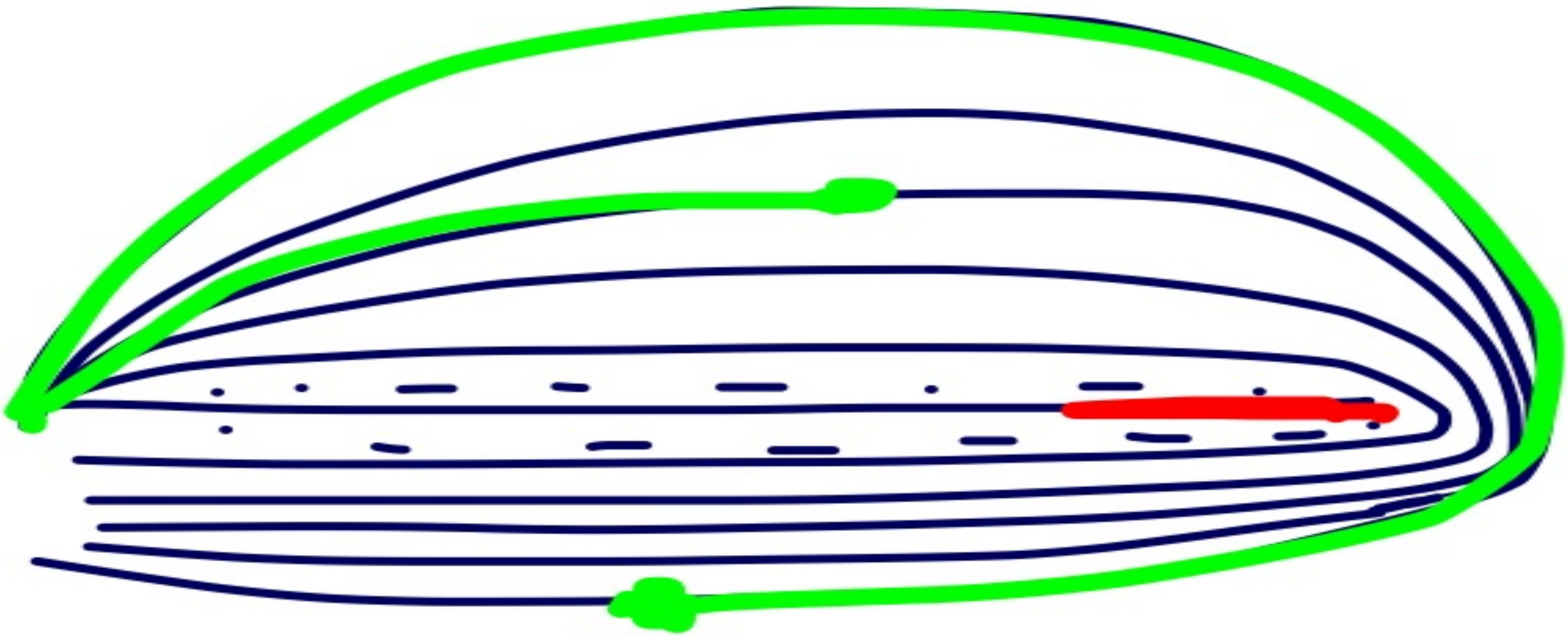
- Let X be a continuum and A a subcontinuum of X with int(A) = ∅.

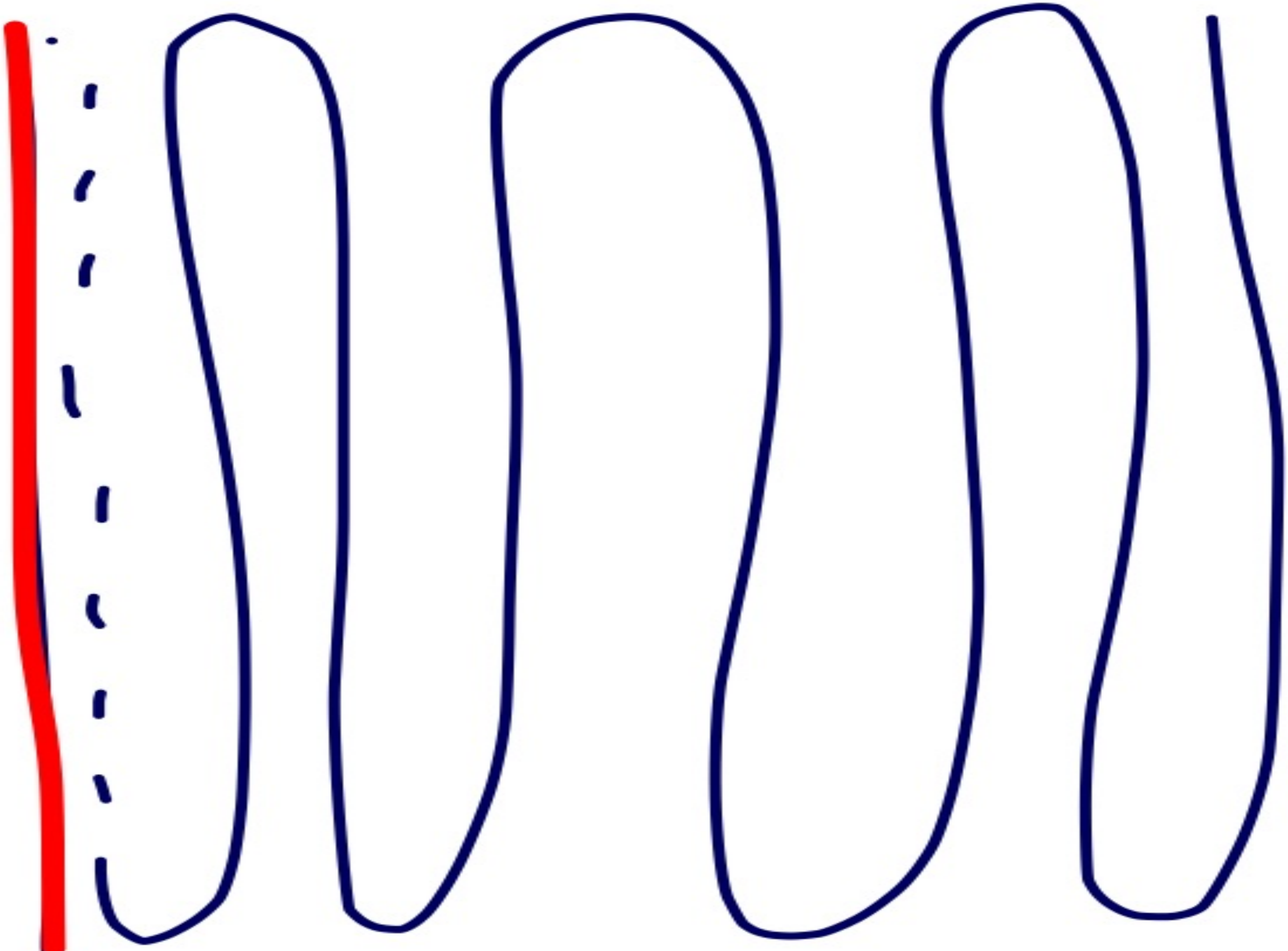
We say that A is **not a weak cut** continuum in X if for any pair of points x,y ∈ X / A there is a subcontinuum M of X such that

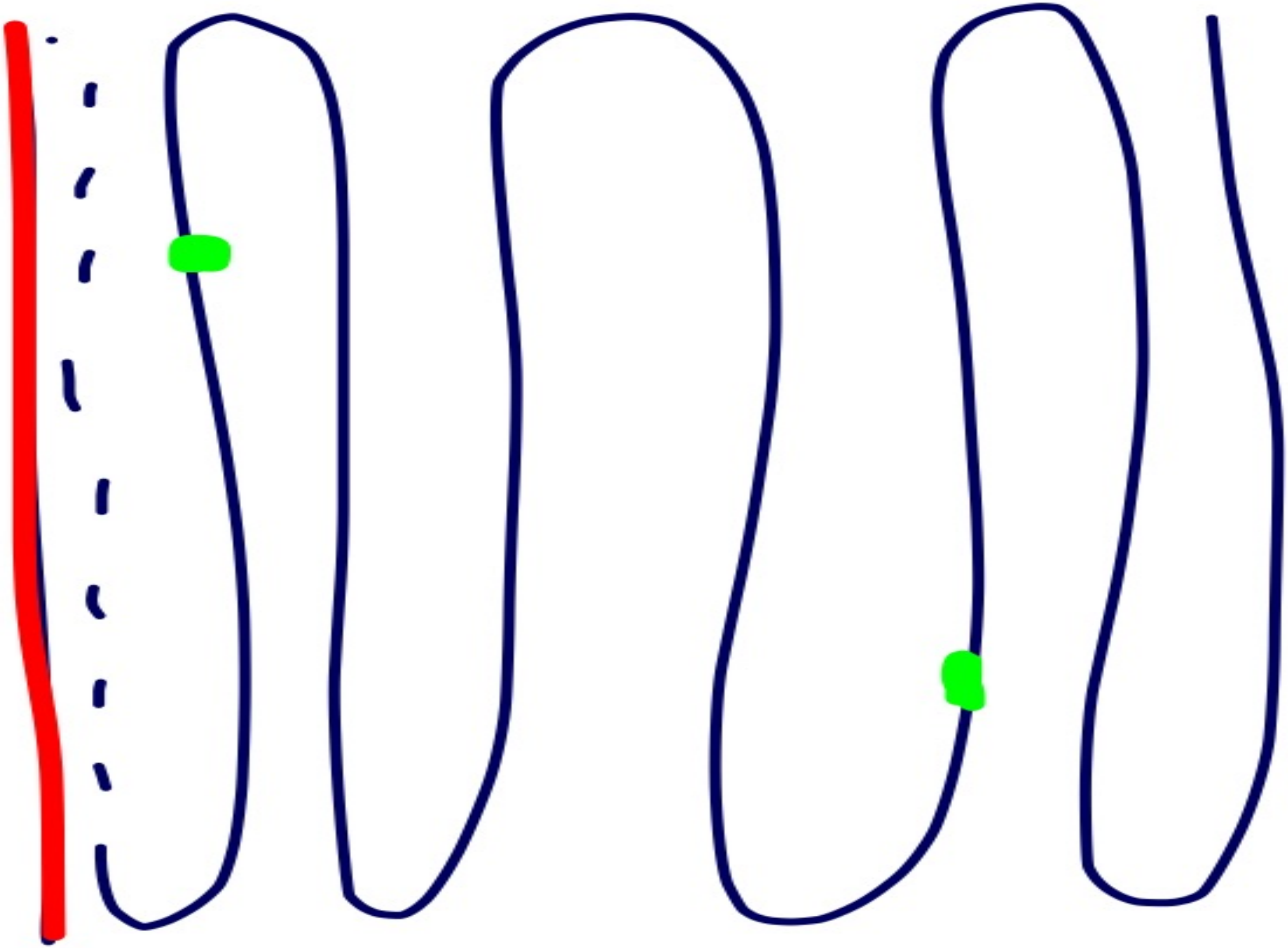
x, y ∈ M and M ∩ A = ∅.

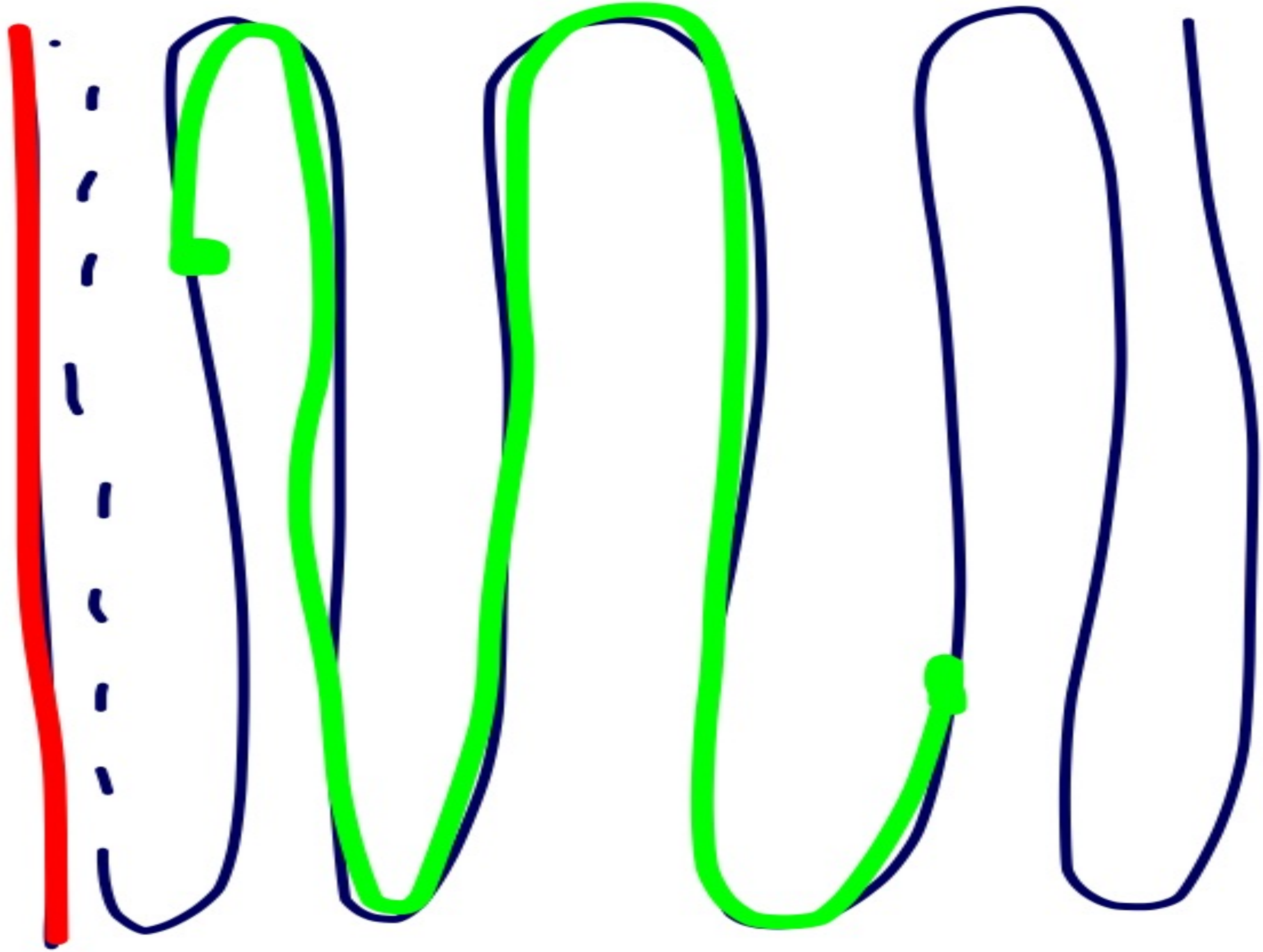


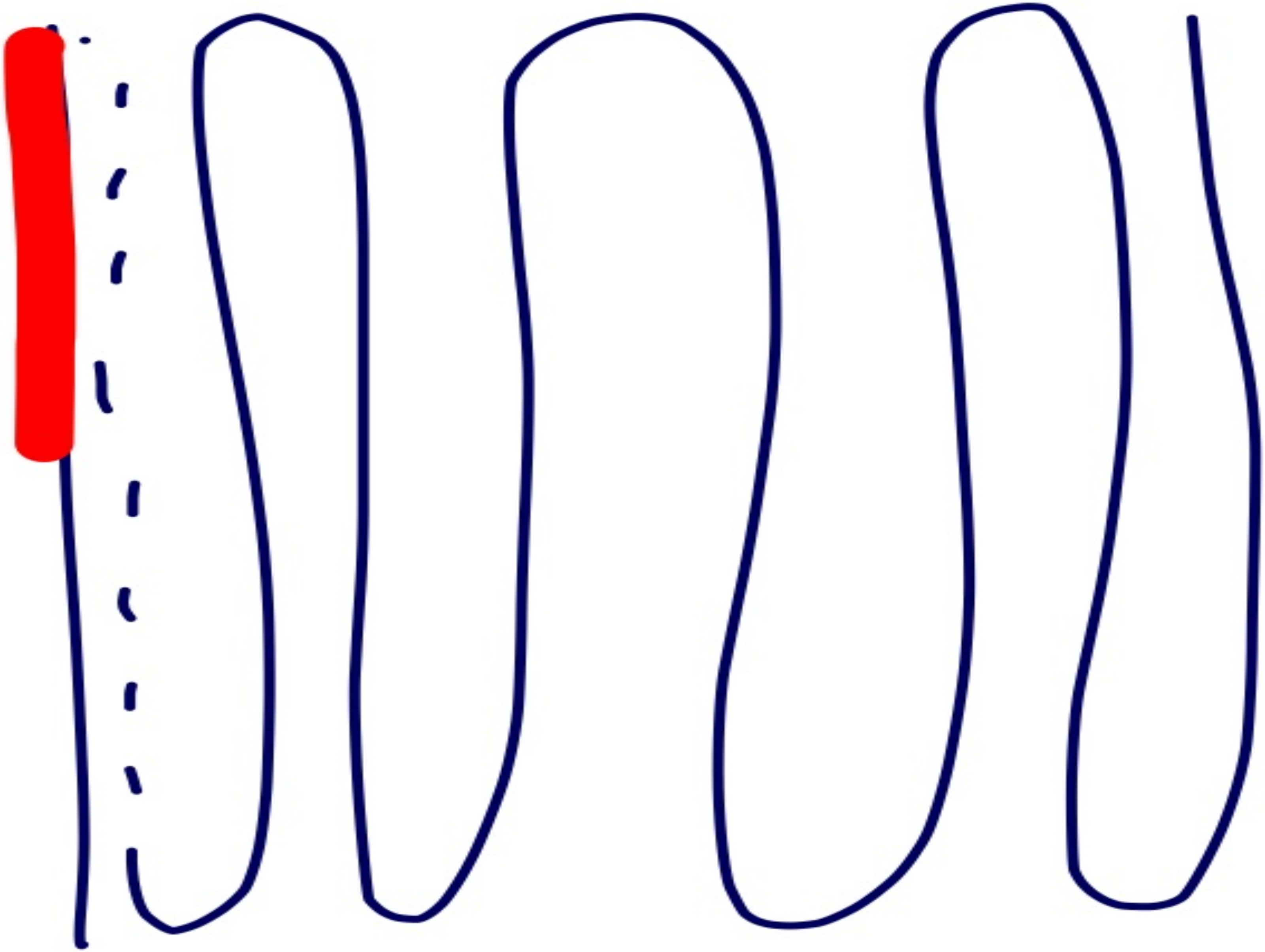


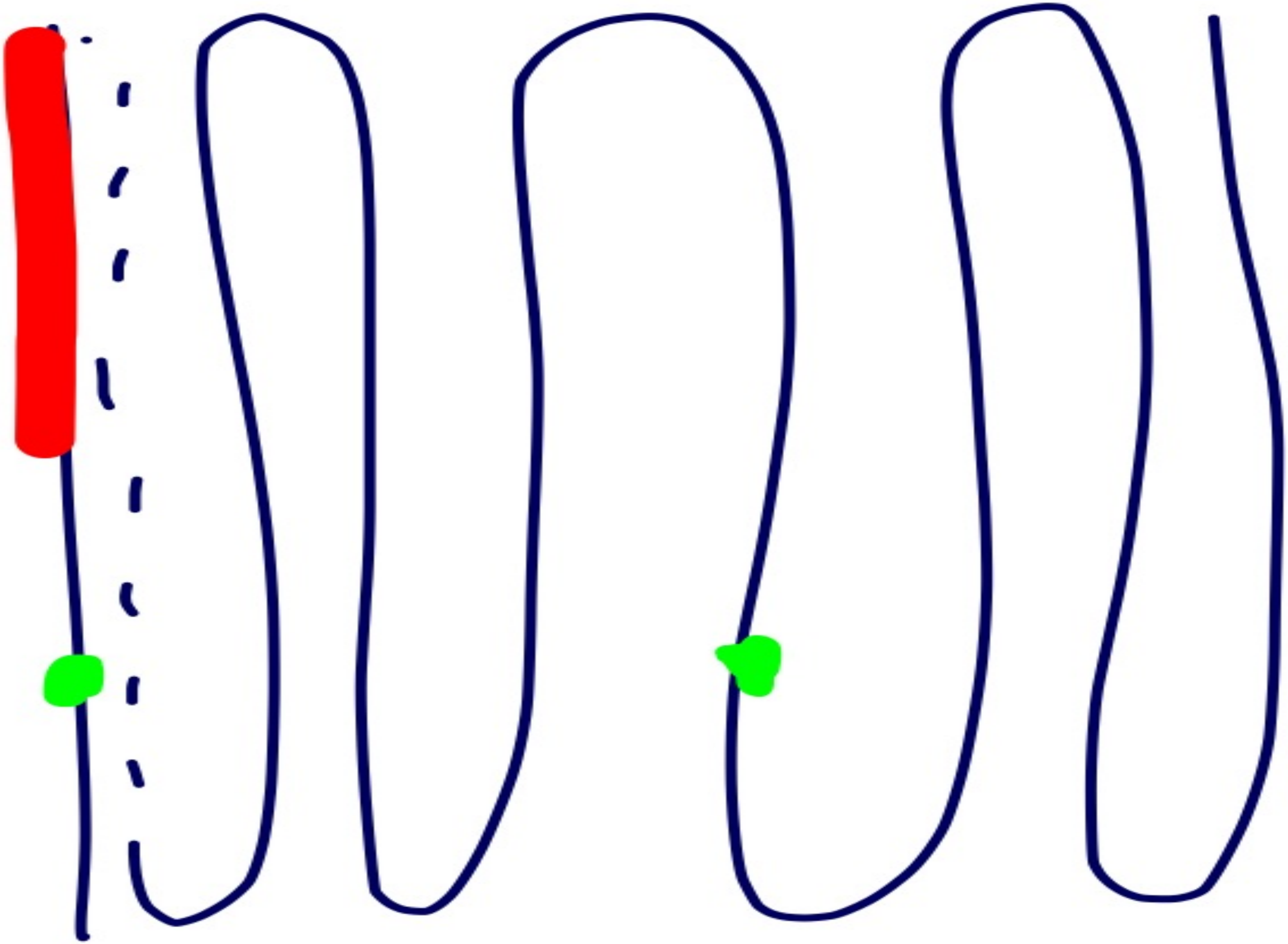


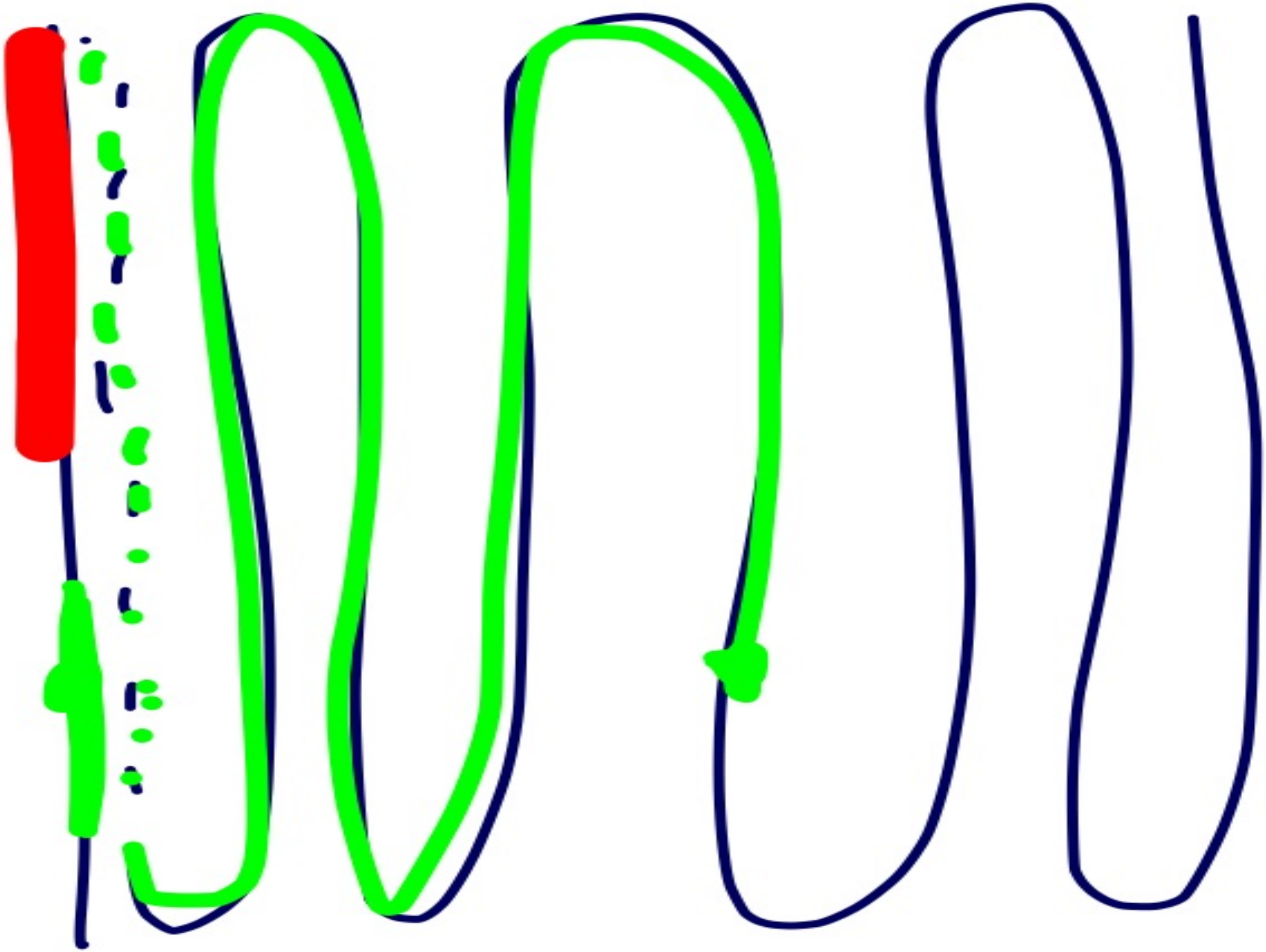


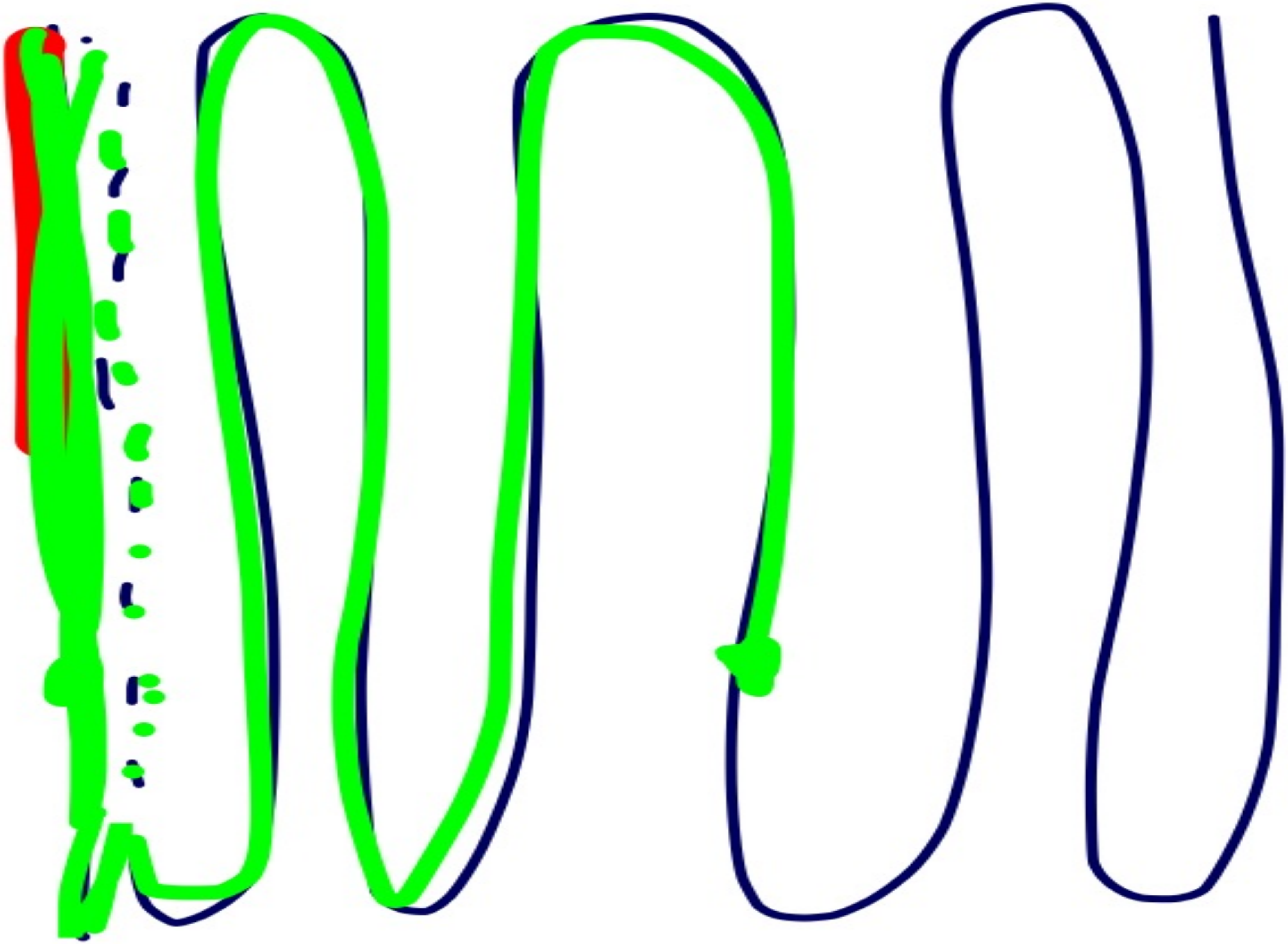












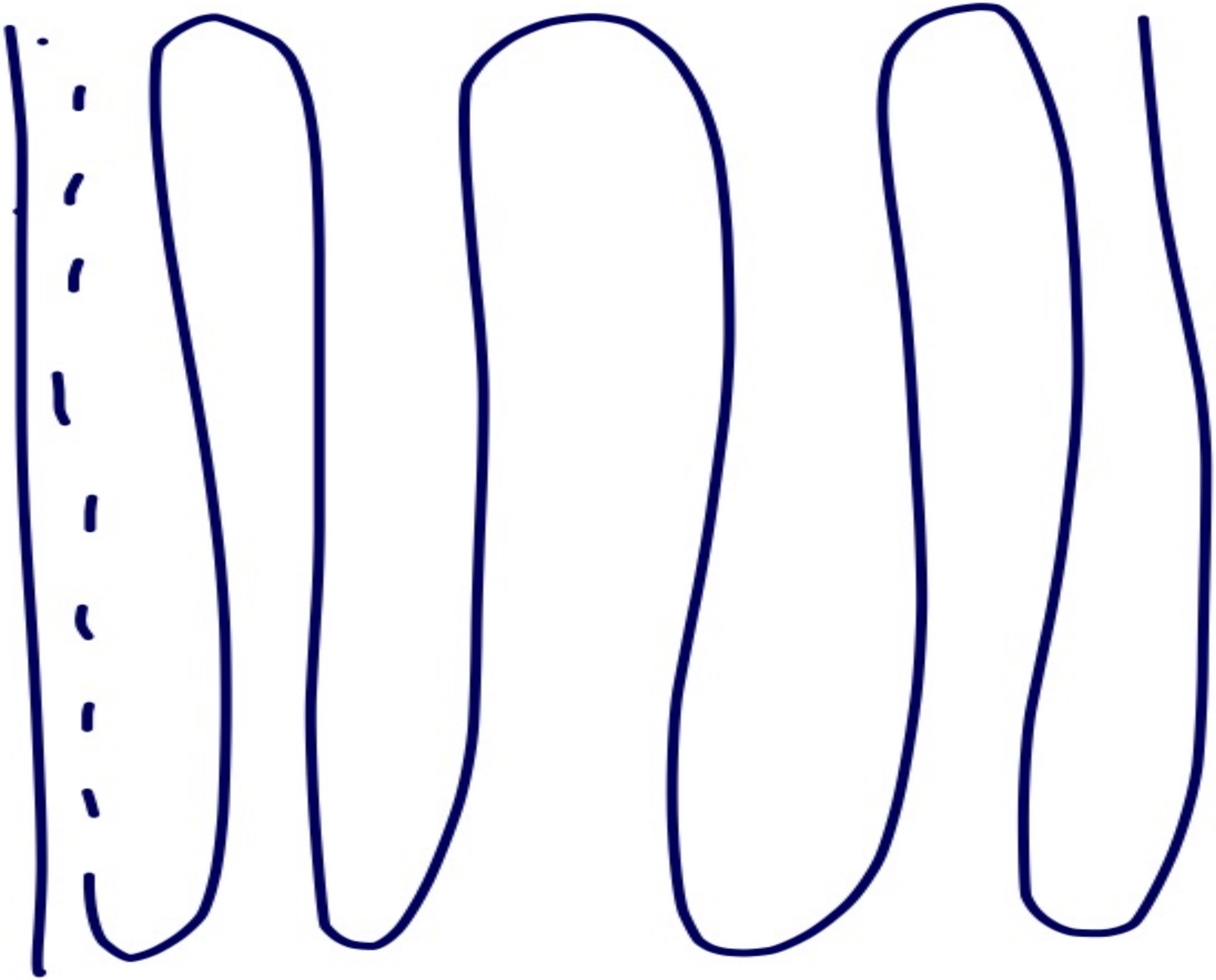
1.5 Non Block

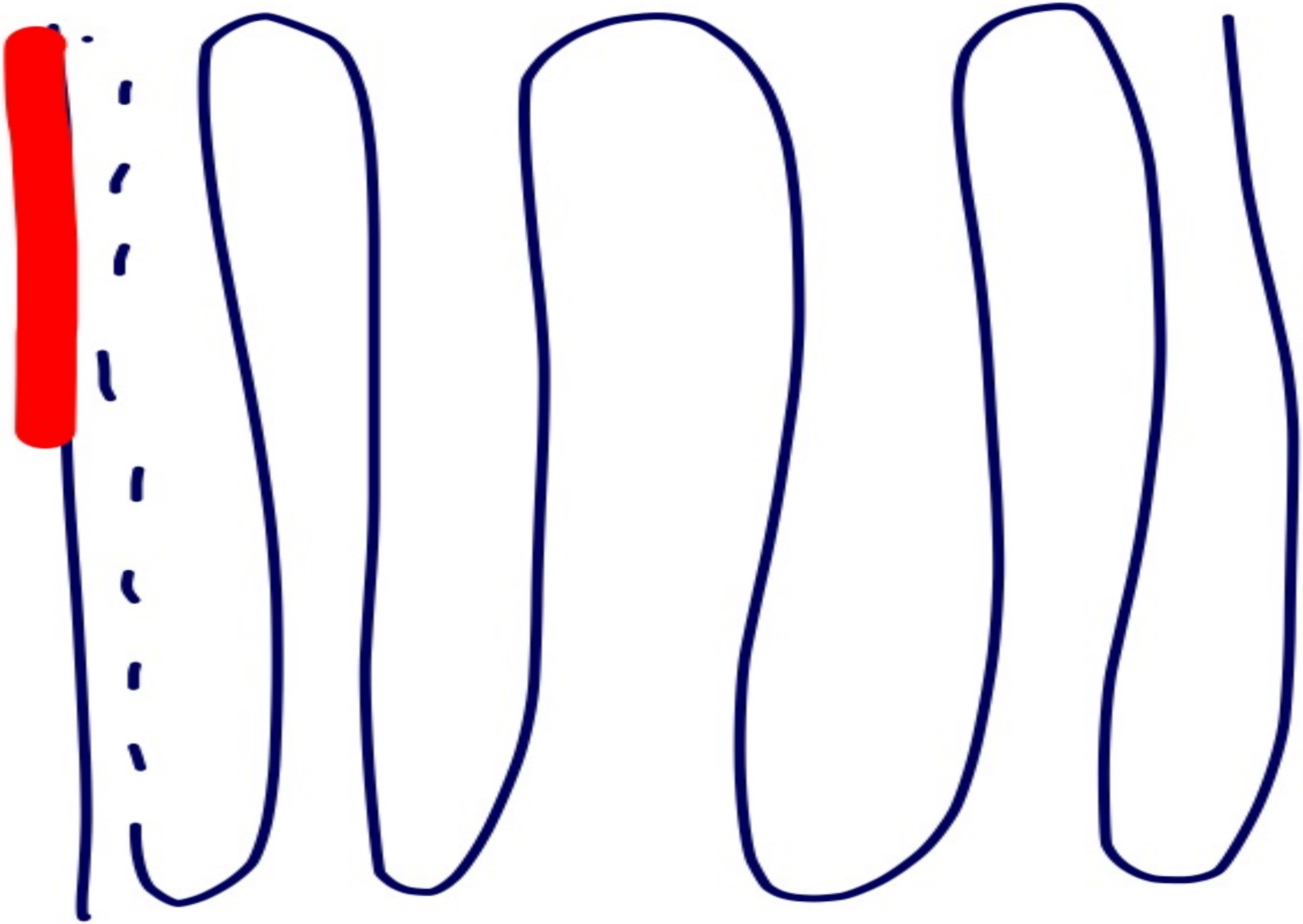
Non Block

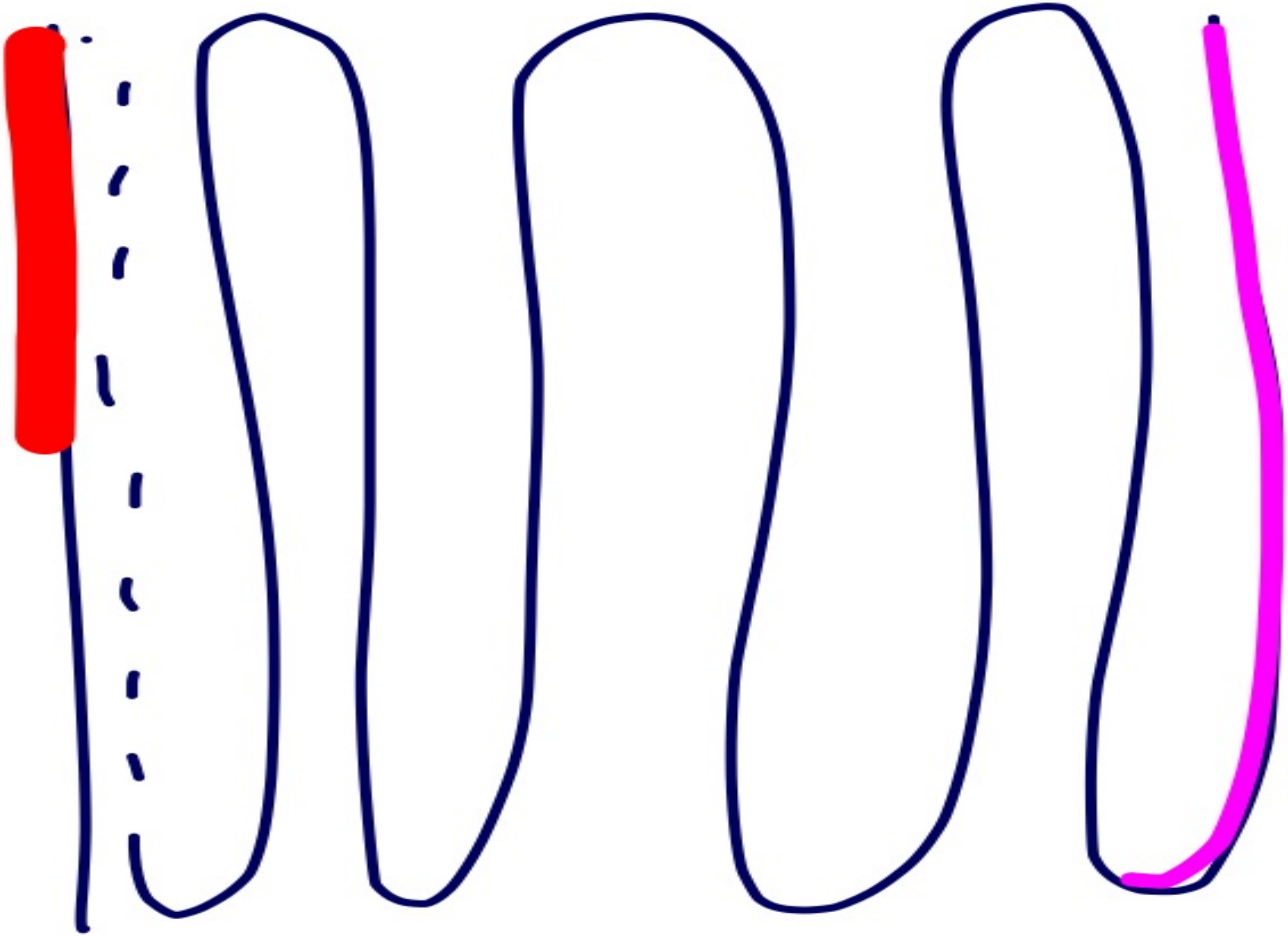
- Let X be a continuum and A a subcontinuum of X with int(A) = $\varnothing$.

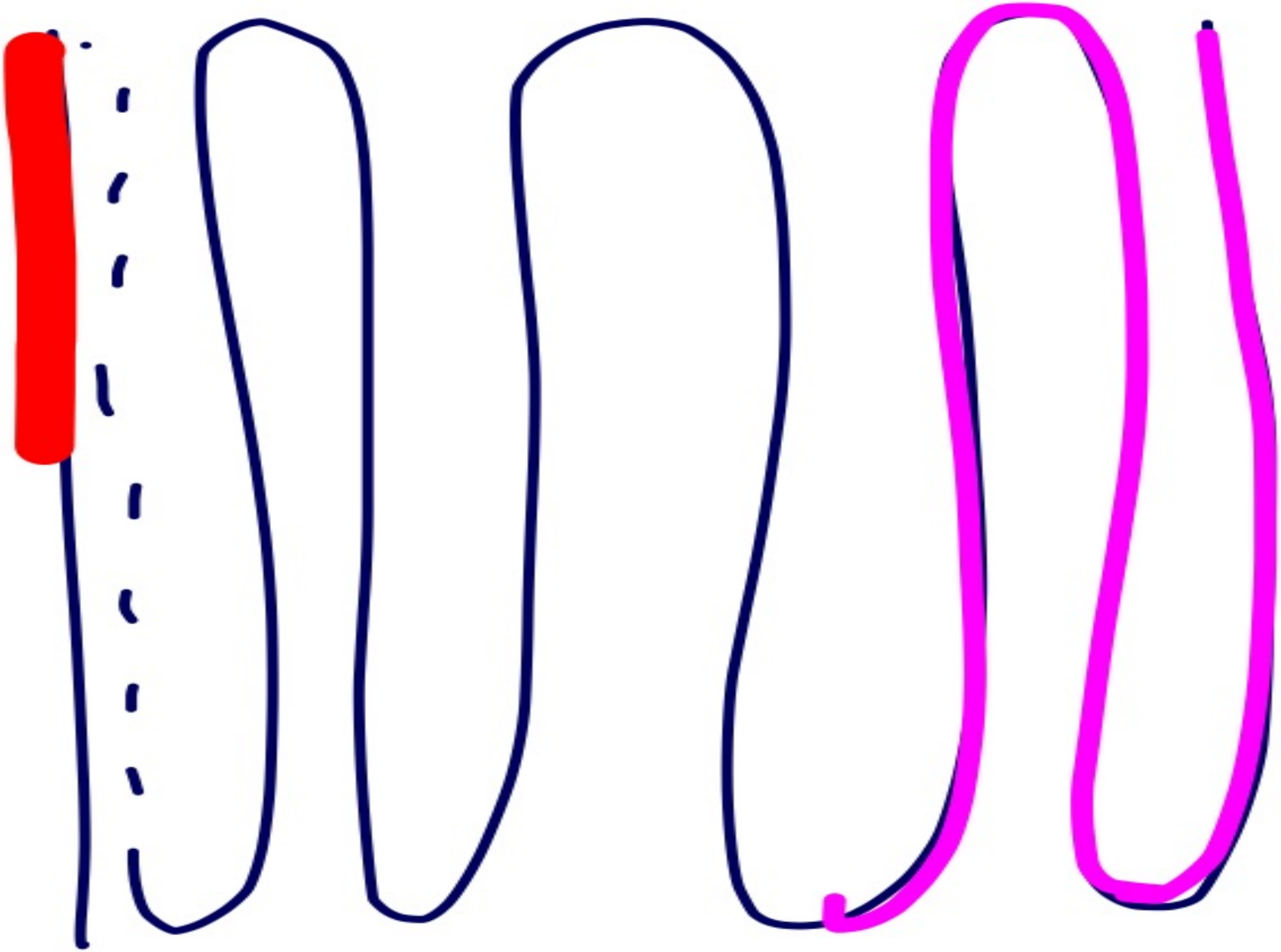
We say that A is **a nonblock** continuum in X provided that there exist a sequence of subcontinua M1,M2,...

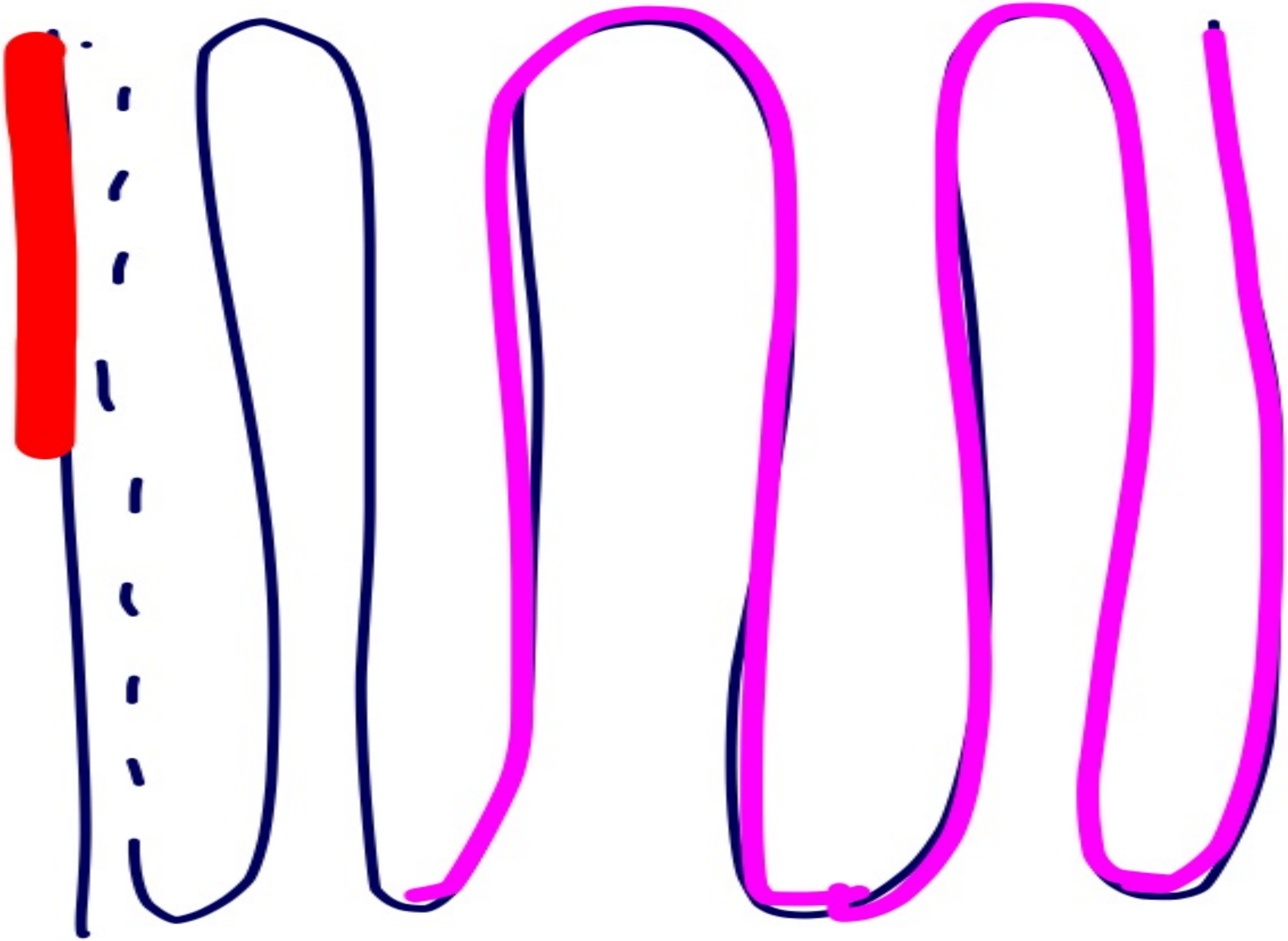
such that $M1 \subset M2 \subset \cdots$ and $\bigcup Mn$ is dense subset of $X \setminus A$.

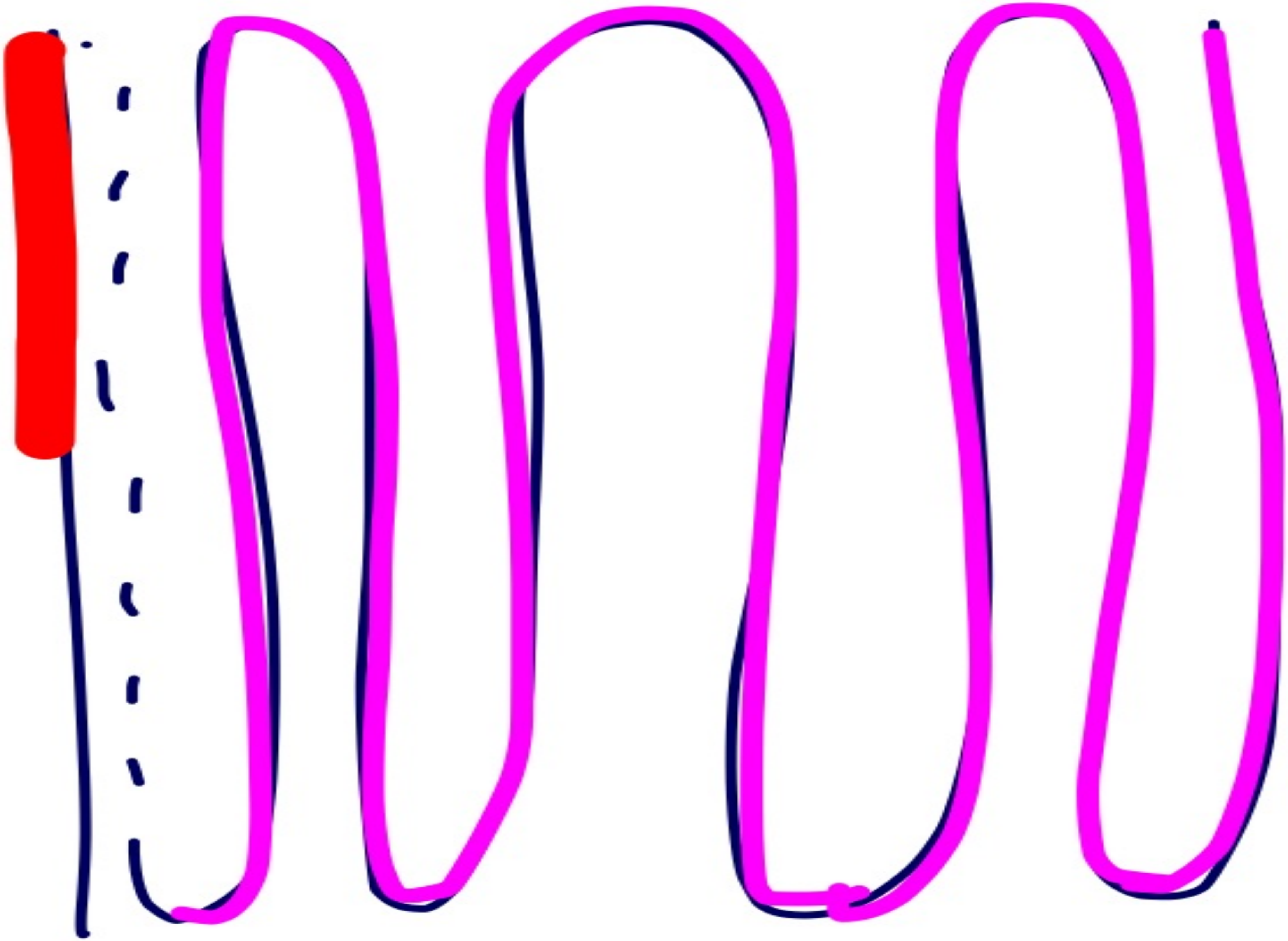


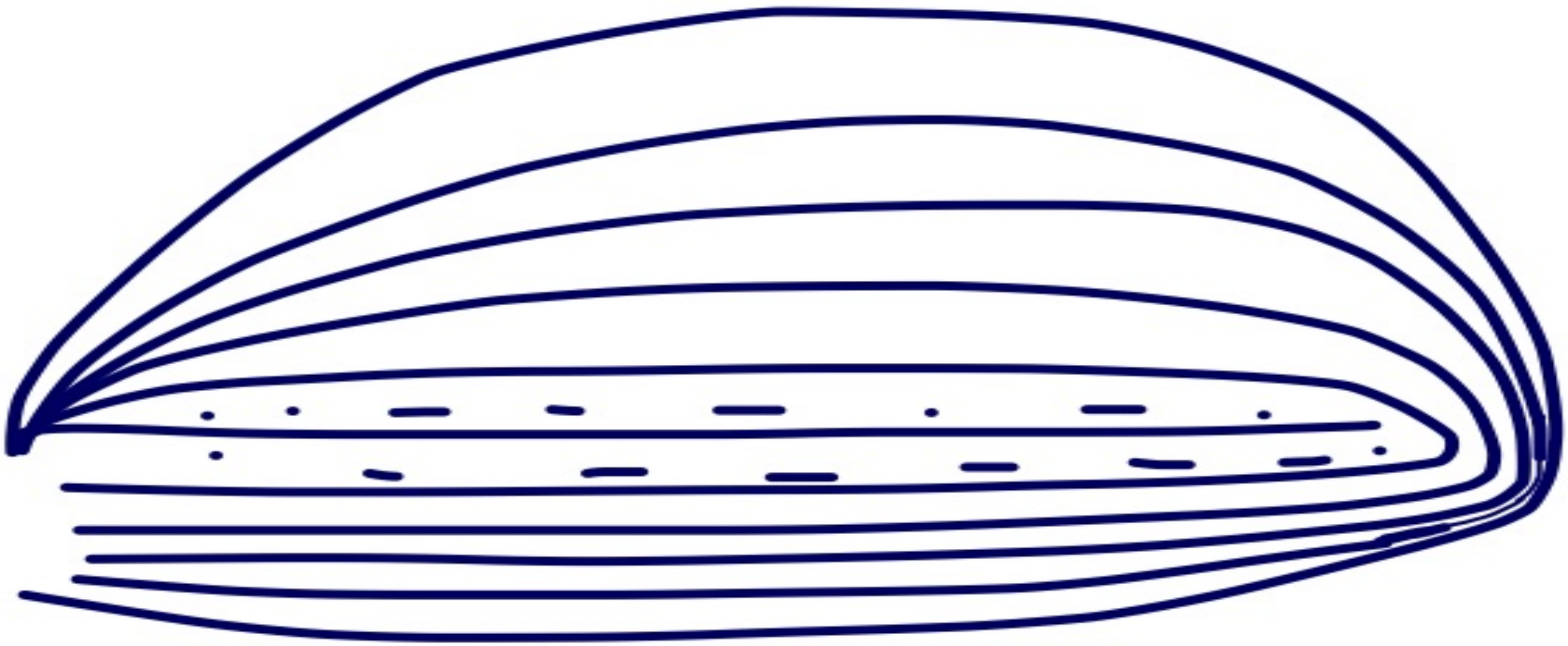


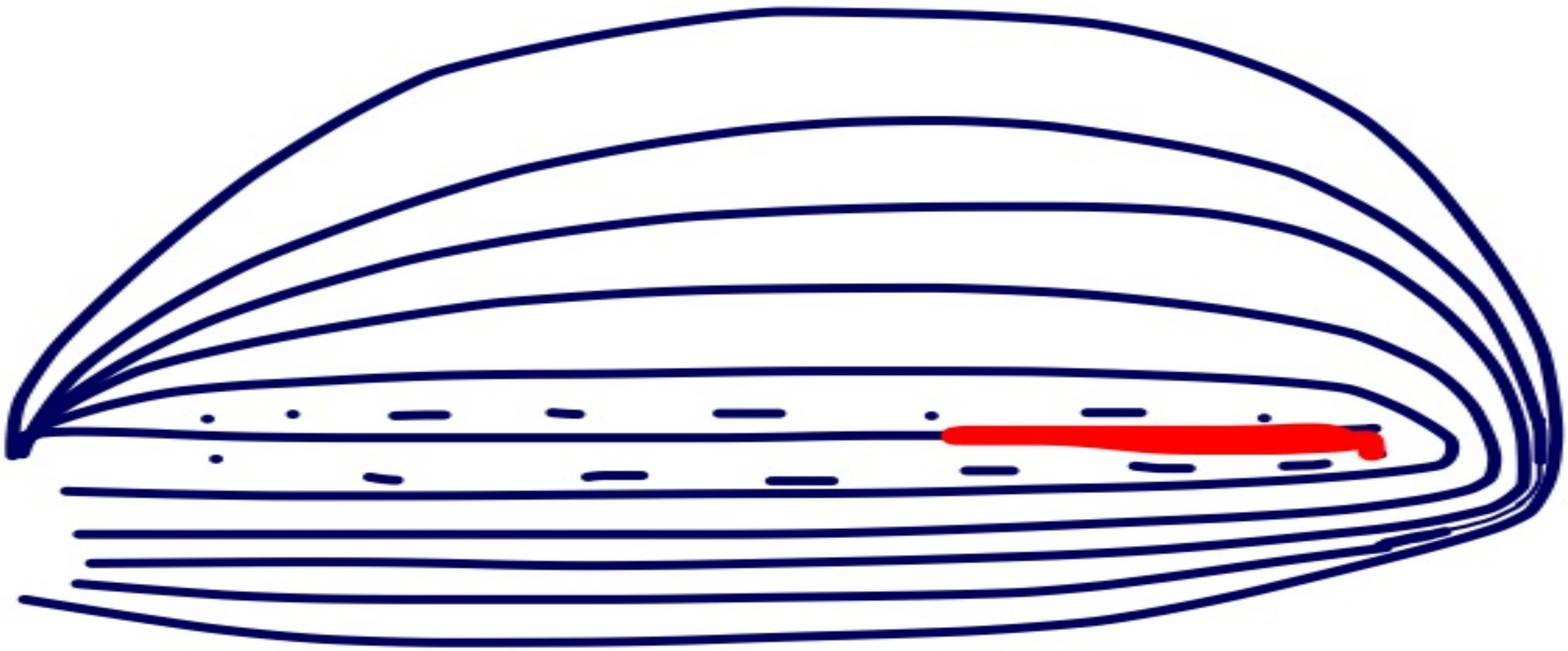


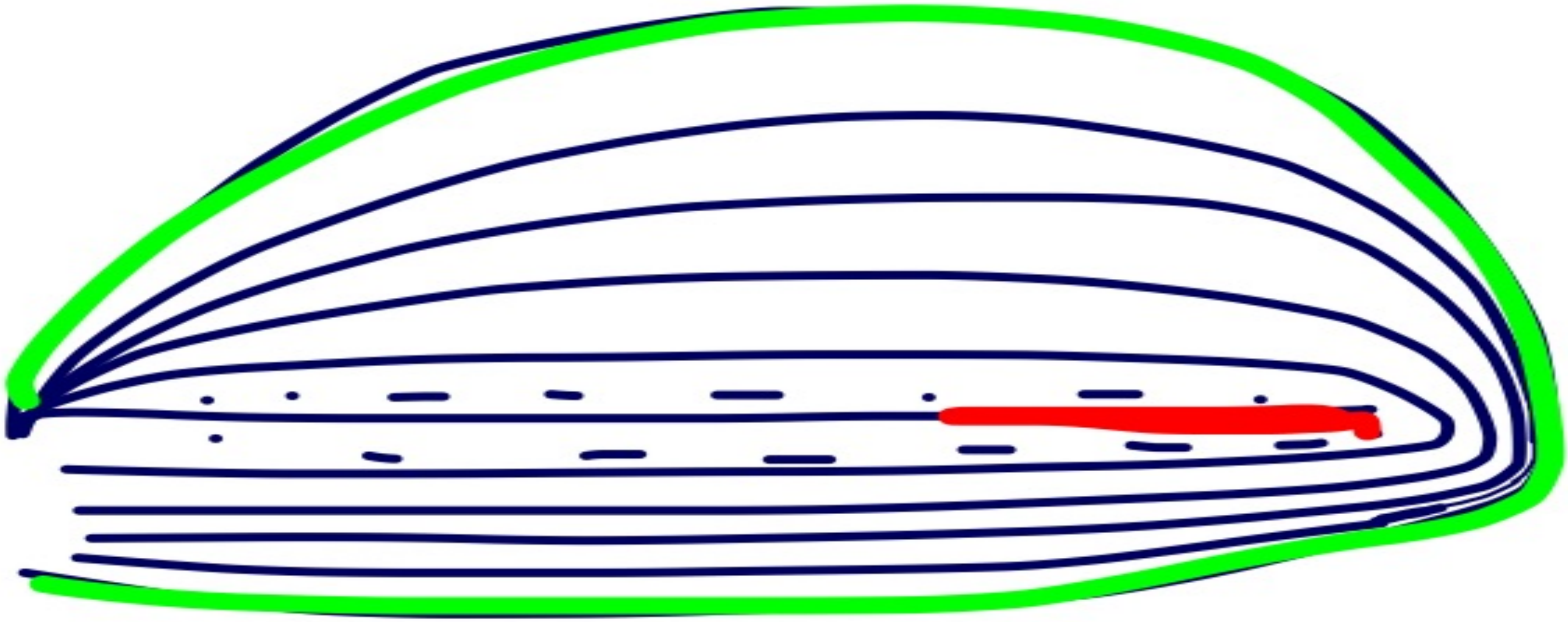


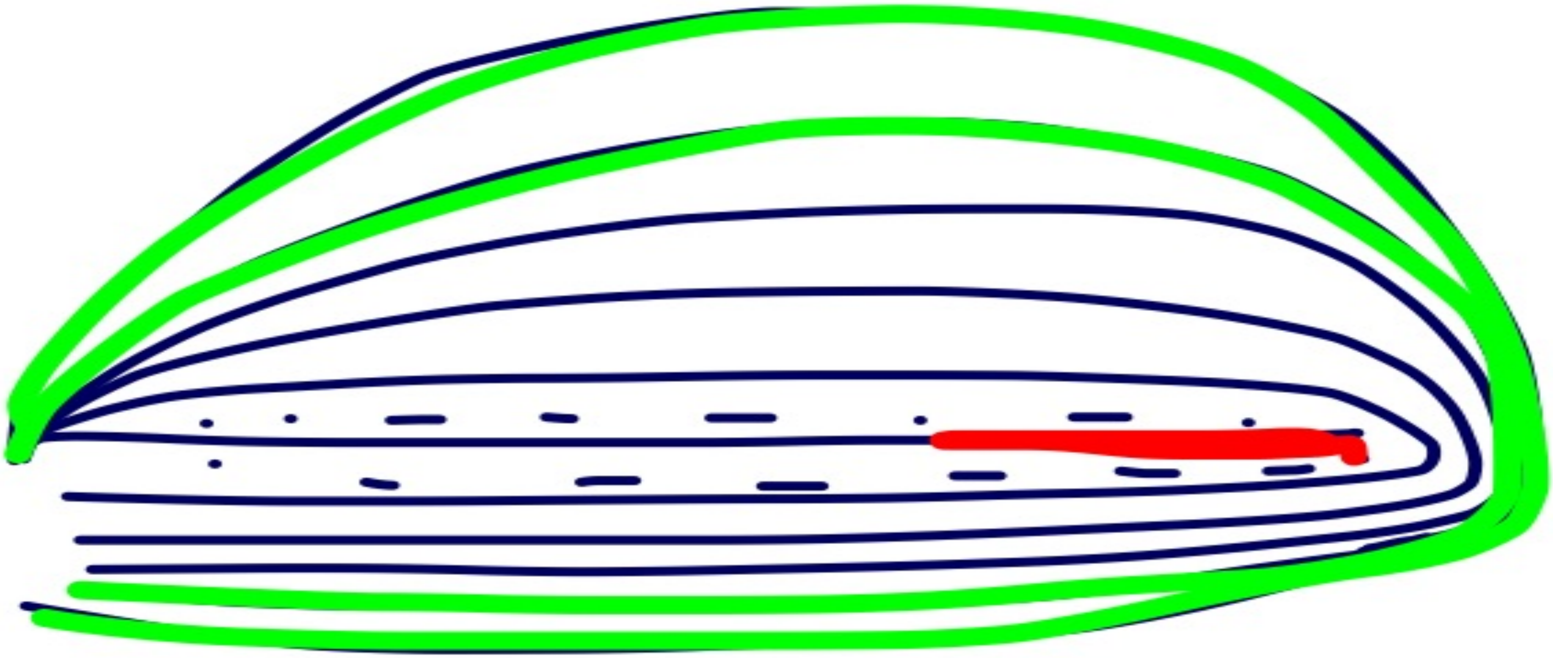


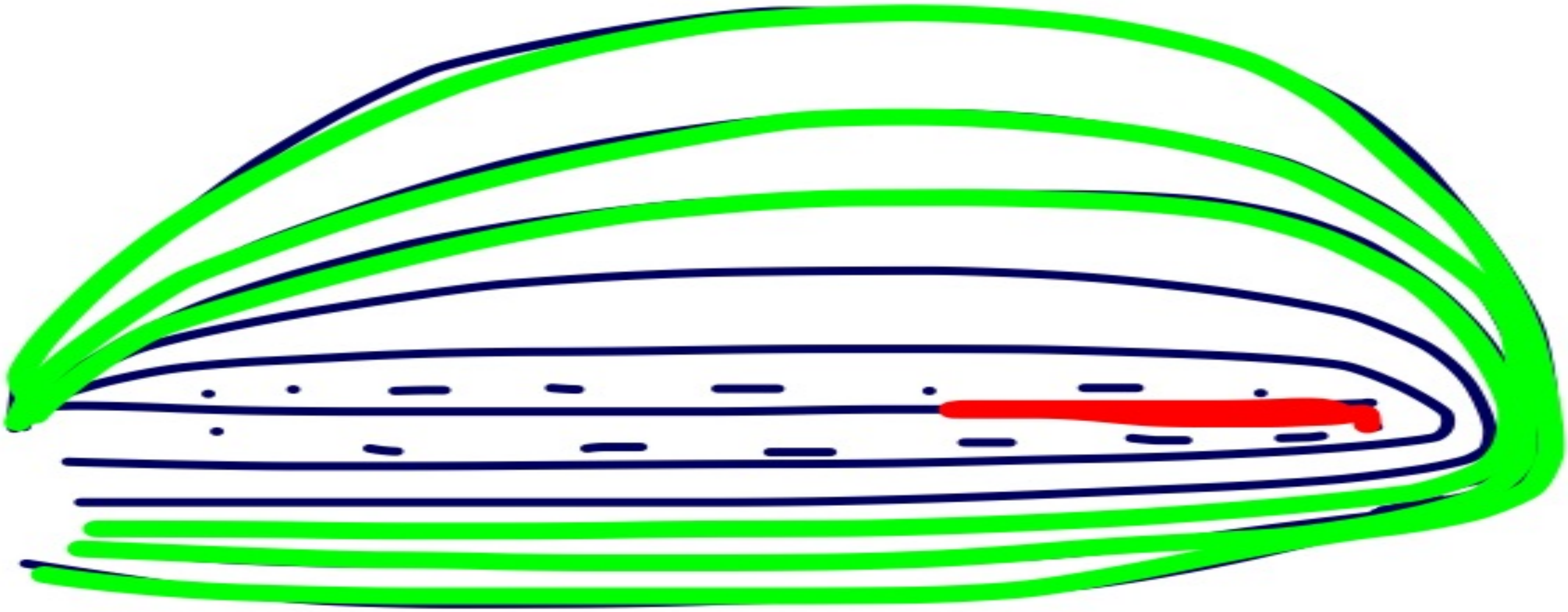


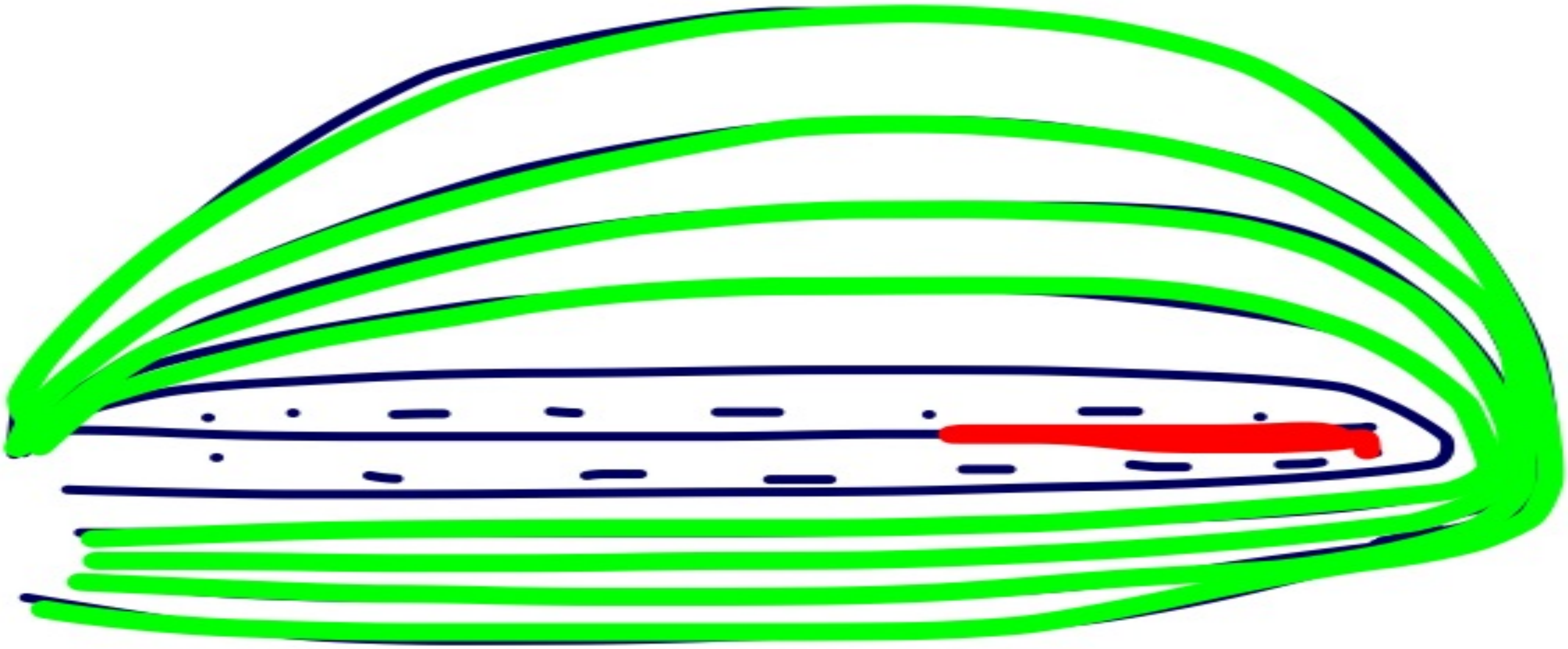


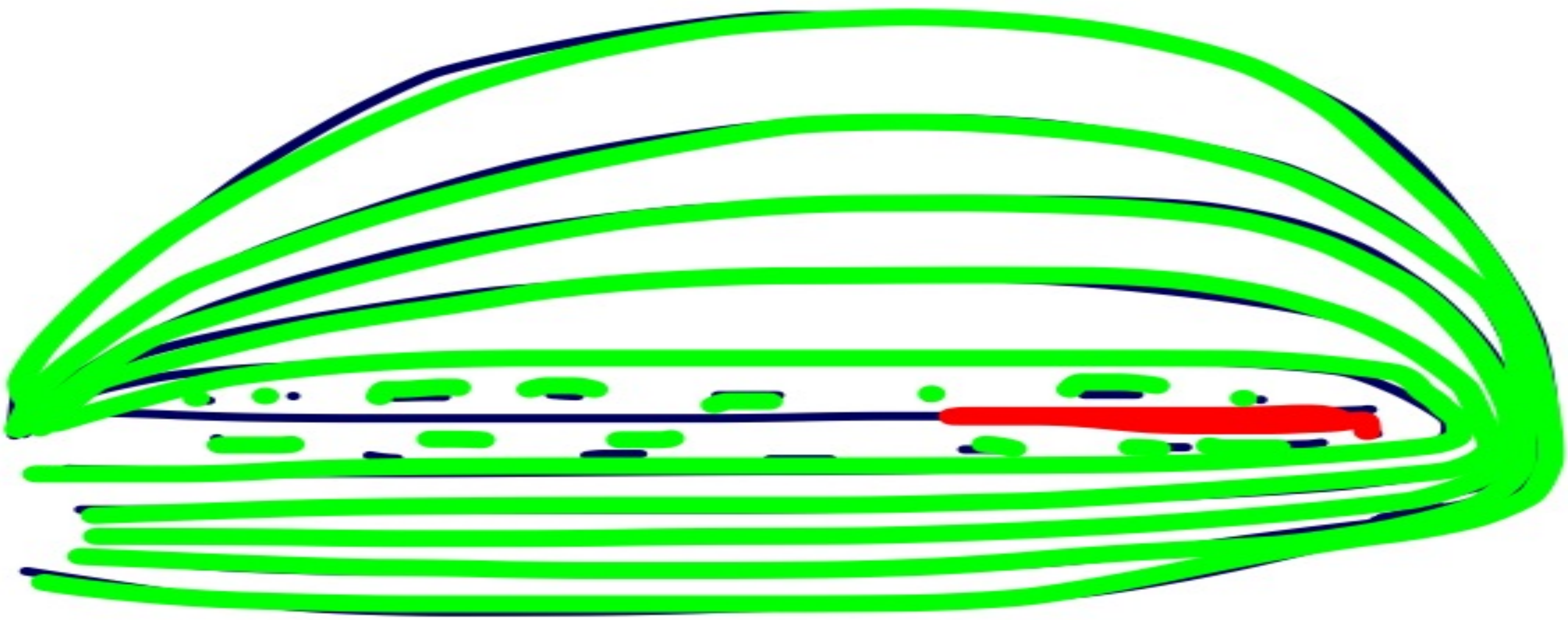












1.6 Shore, Not strong Center and Non Cut

Shore, Not strong Center and Non Cut

- Let X be a continuum and A a subcontinuum of X with int(A) = ∅. We say that A is:

a shore continuum in X if for each $\varepsilon > 0$ there is a subcontinuum M of X such that $H(M, X) < \varepsilon$ and $M \cap A = \emptyset$.

Shore, Not strong Center and Non Cut

- (5) **not a strong center** in X provided that for each pair of nonempty open subsets U and V of X there exists a subcontinuum M of X such that $M \cap U \neq \varnothing \neq M \cap V$ and $M \cap A = \varnothing$.
- (6) **a noncut** continuum in X if $X \setminus A$ is connected.

2. Previous Results

Theorem
J. Bobok, P. Pyrih and B. Vejnar

- Colocally Connected => non- weak cut => non-block => shore => strong center => non cut
- Non cut ≠>strong center ≠>shore≠>non block ≠> non weak cut ≠> colocally connected
- If X is locally connected then Colocally Connected <=> non- weak cut <=> non-block <=> shore< => strong center< => non cut

$F_1(X)$ is a sumbcontinuum in $\mathcal{H}(X)$

- Where $\mathcal{H}(X)$ is any Hyperspace,

  $\mathcal{H}(X) \in \{ 2^X, F_n(X), C(X), C_n(X) \}$

In fact $F_1(X)$ has empty interior in $\mathcal{H}(X)$

$F_1(X)$ is homeomorphic to X.

So we want to know if $F_1(X)$ is a :

- Collocal connected
- Non weak cut
- Non block
- Shore
- Strong center
- Non cut

subcontinum in $\mathcal{H}(X)$.

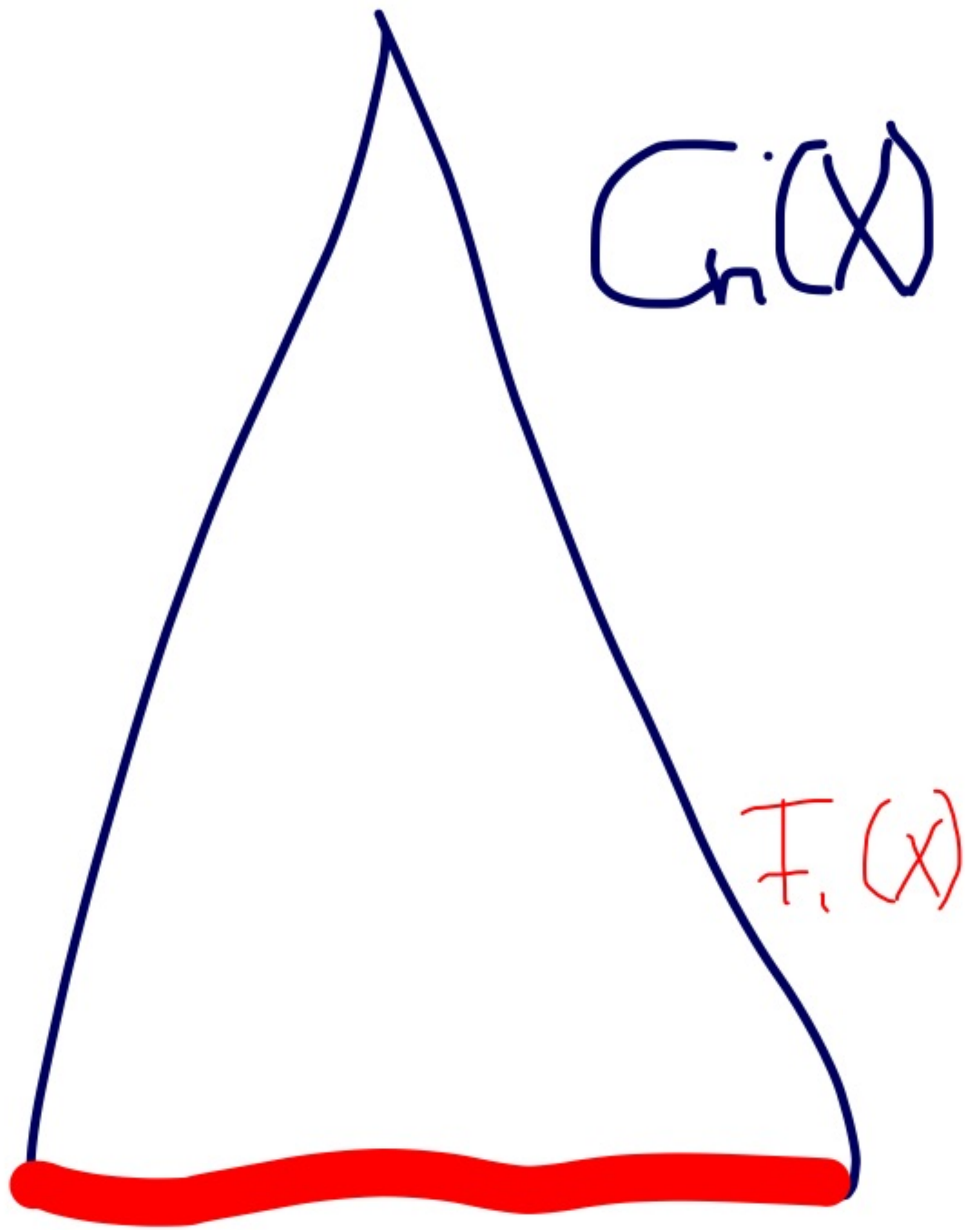
3 Main Result in the Hyperspace Cn(X)

Theorem (VMV and JMM, 2017)

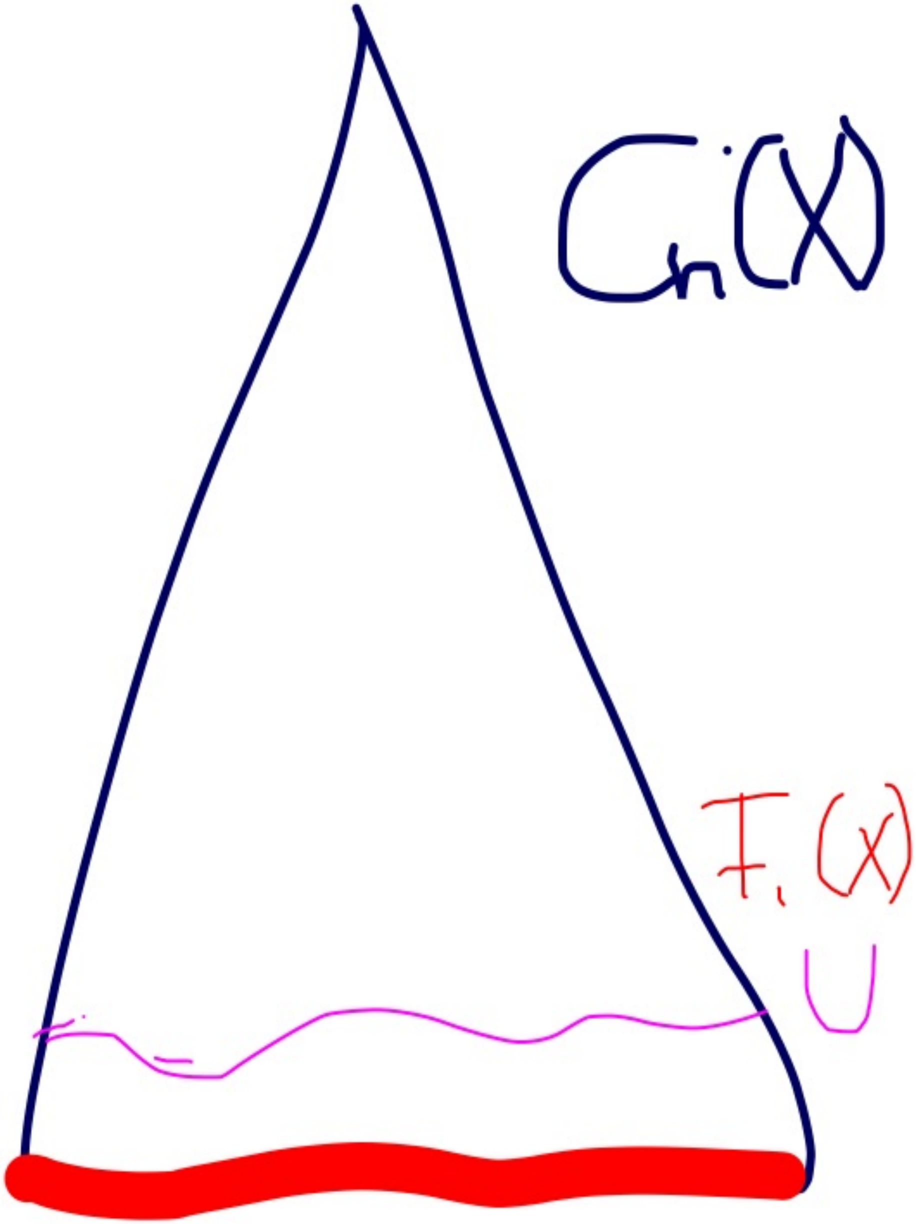
- For every continuum X and each positive integer n, $F_1(X)$ is a colocally connected subcontinuum in $C_n(X)$

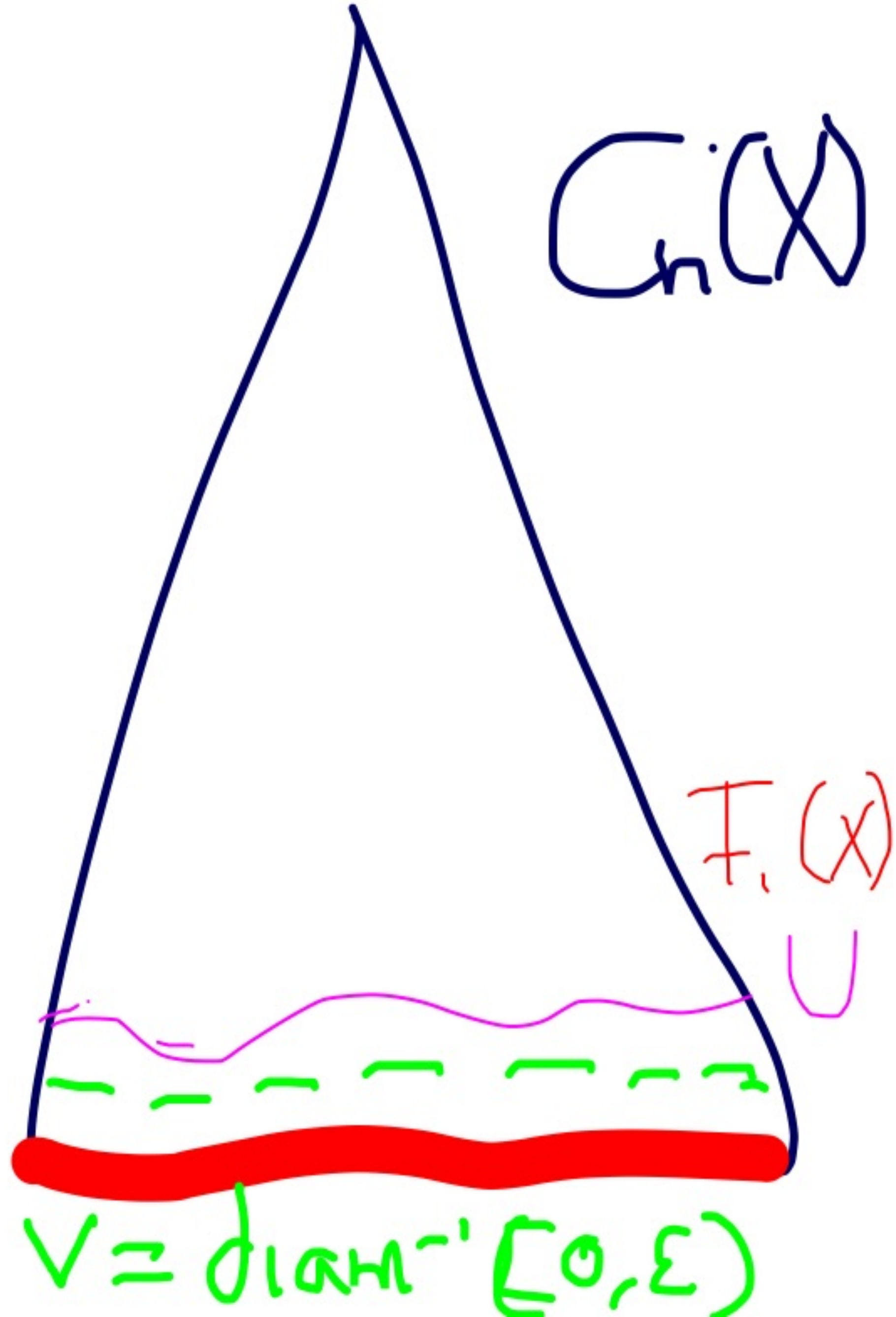
ORDERED ARCS IN HYPERSPACES

- Given the Hyperspace $C_n(X)$ if $A,B \in C_n(X)$ and $A \subseteq B$ we define an ordered arc from A to B in $C_n(X)$ is a map $\alpha:[0,1] \to C_n(X)$ such that:
- $\alpha(0)=A$, $\alpha(1)=B$ and
- if $0 \leq s < t \leq 1$ then $\alpha(s) \subseteq \alpha(t)$

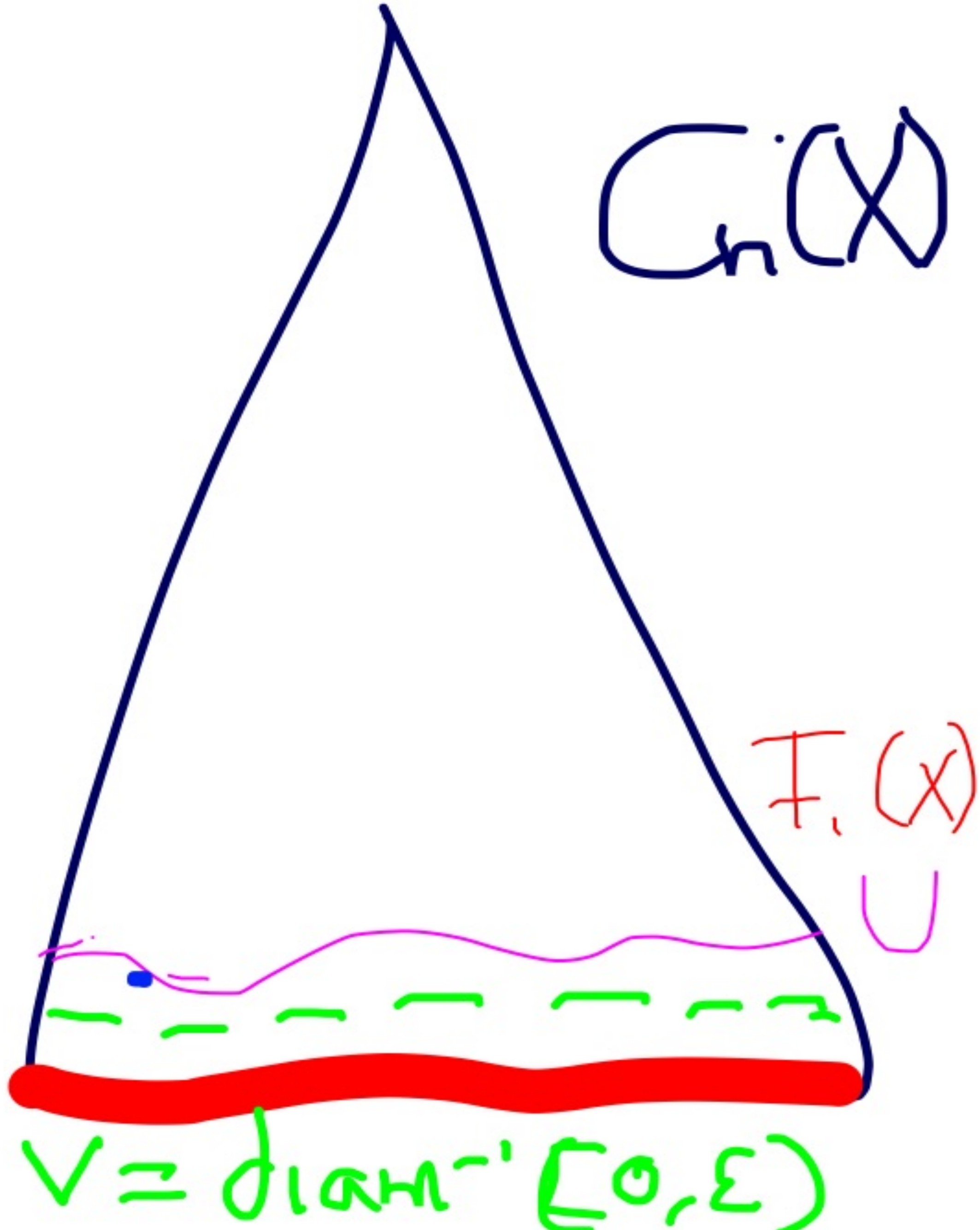


$C_n(X)$
$F_1(X)$

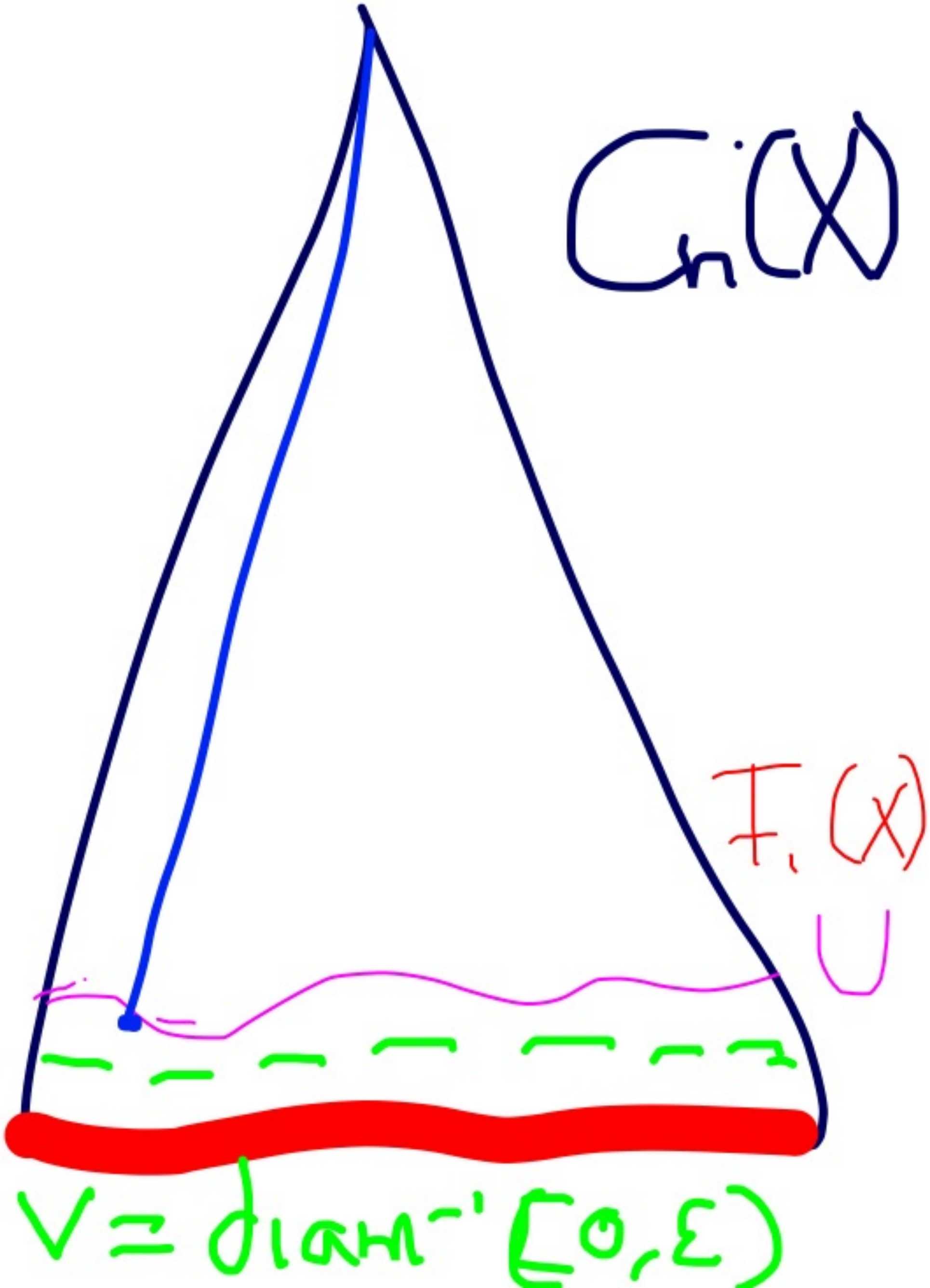

$C_n(X)$
$F_1(X)$
$\cup$



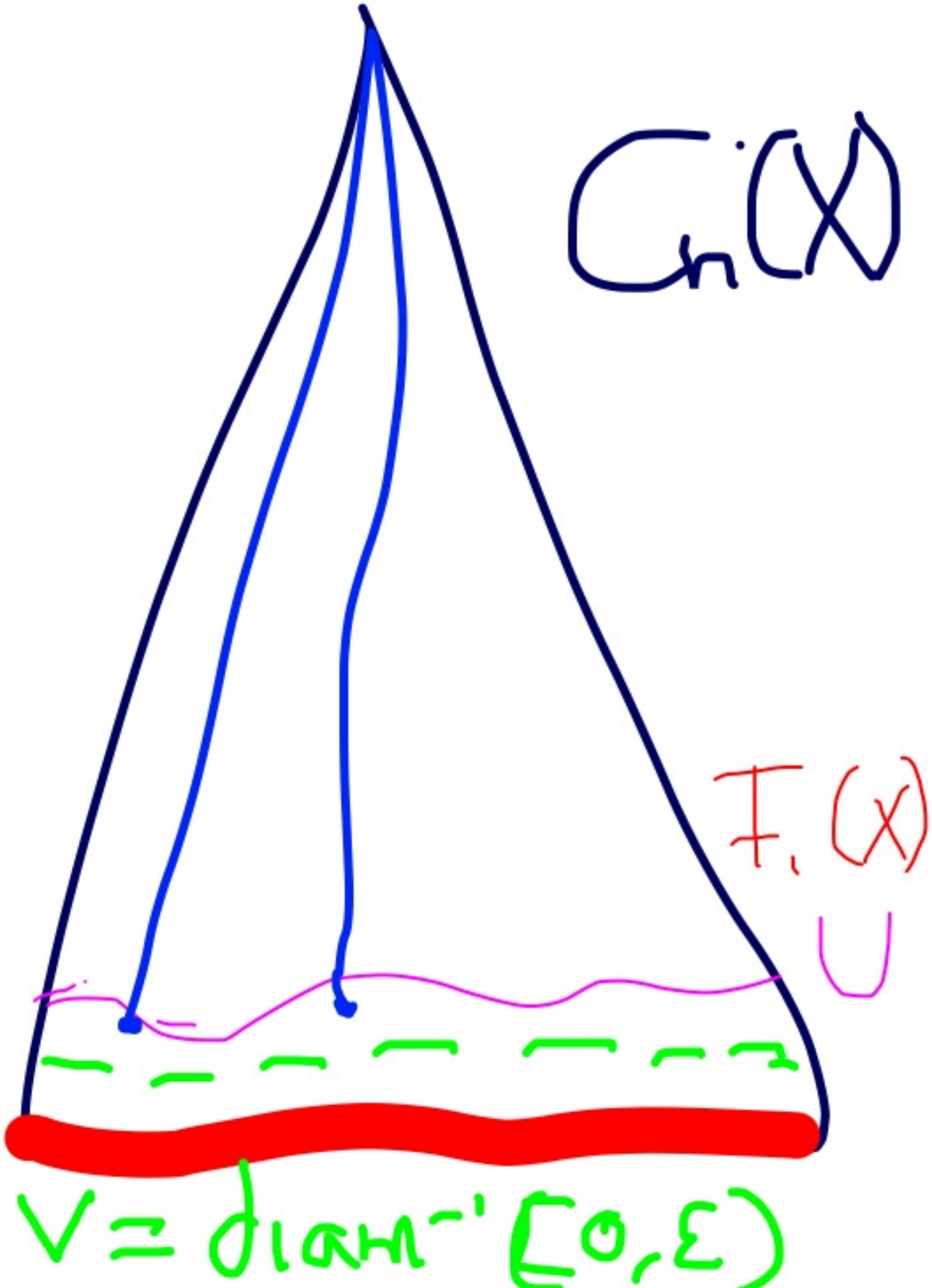
$C_n(X)$
$F_1(X)$
U
$V = \mathrm{diam}^{-1}[0,\varepsilon)$



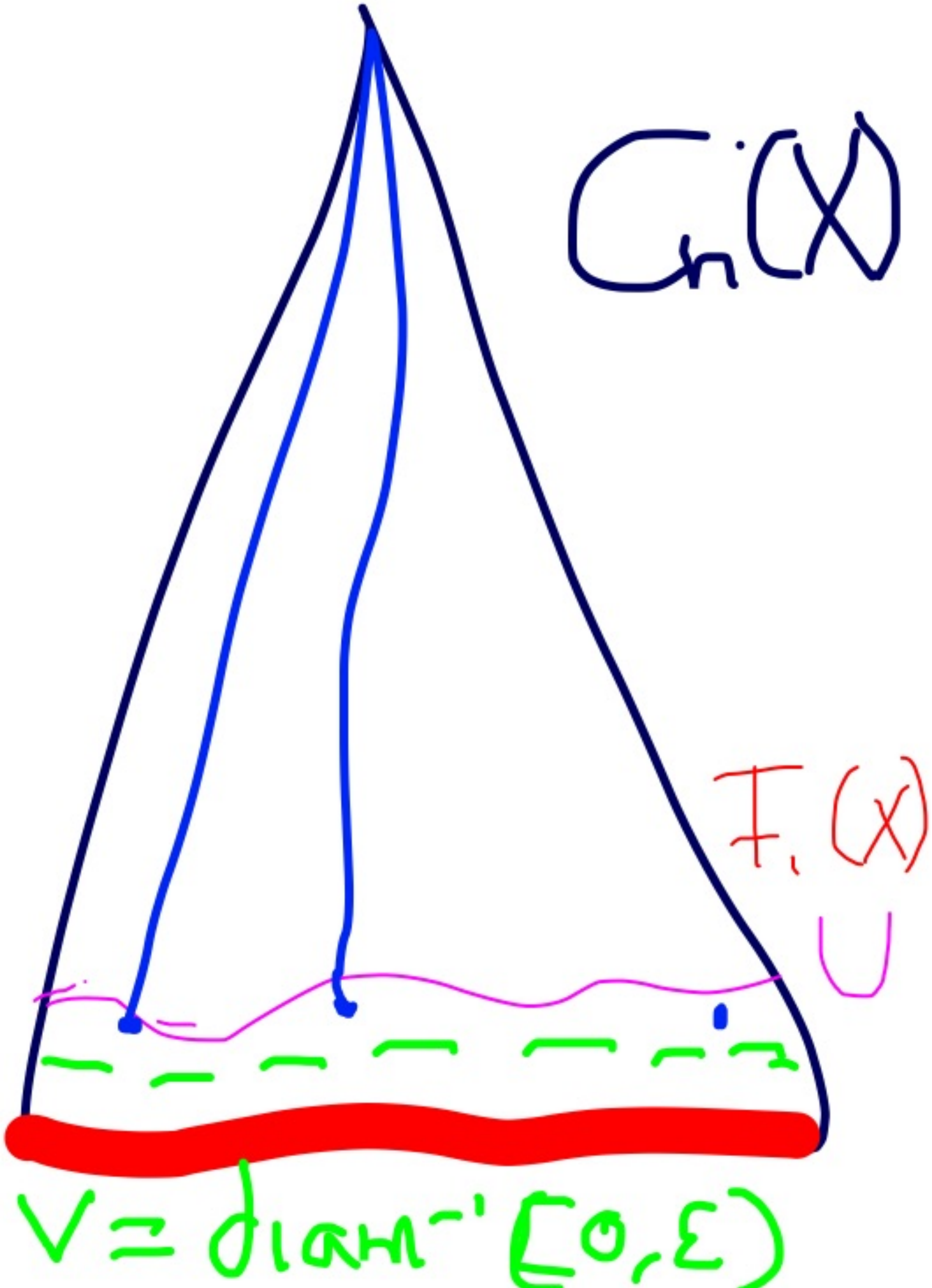
$C_n(X)$
$F_1(X)$
U
$V = \text{diam}^{-1}[0, \varepsilon)$



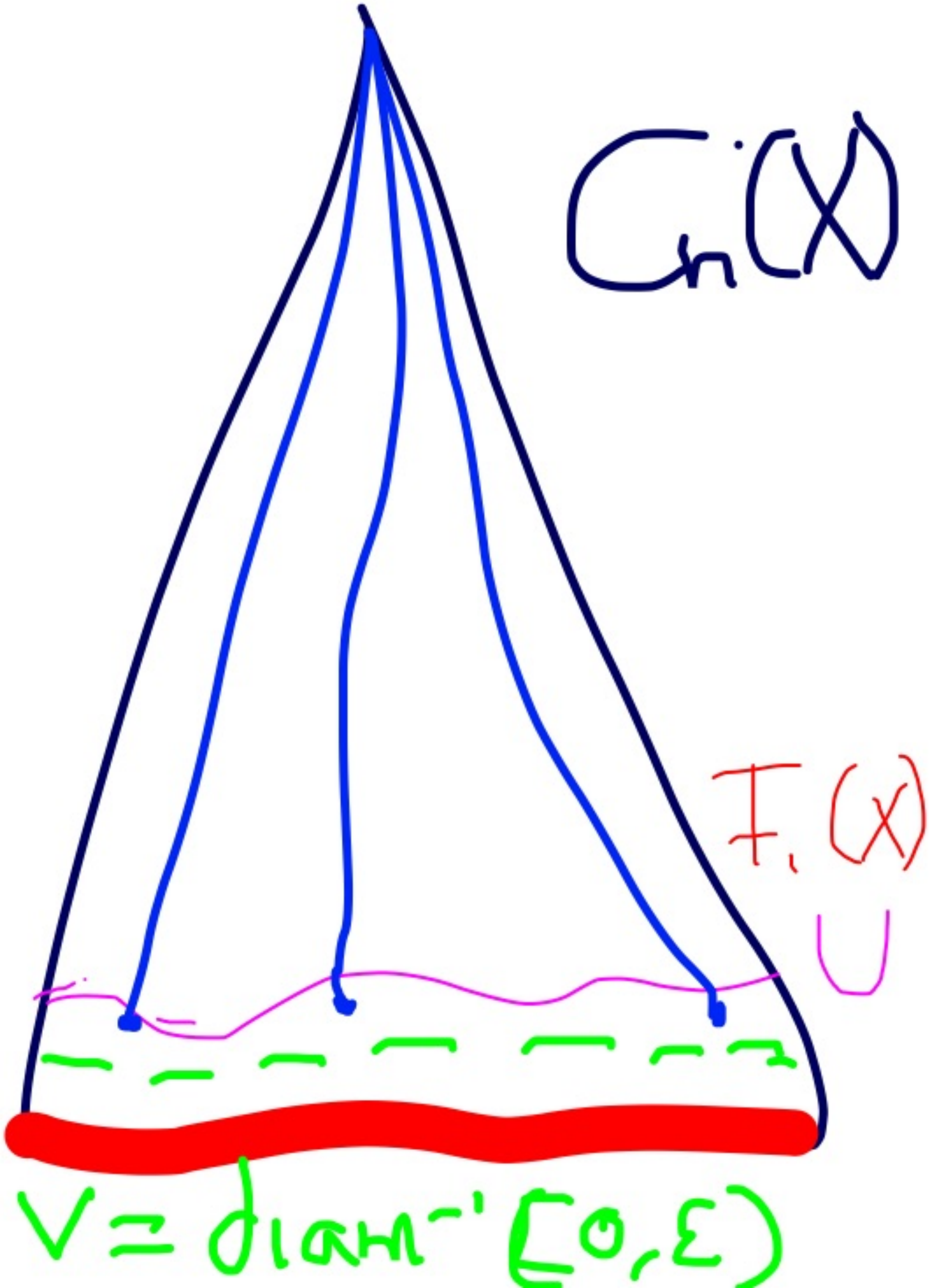
C_n(X)
F_1(X)
∪
V = diam^{-1}[0, ε)

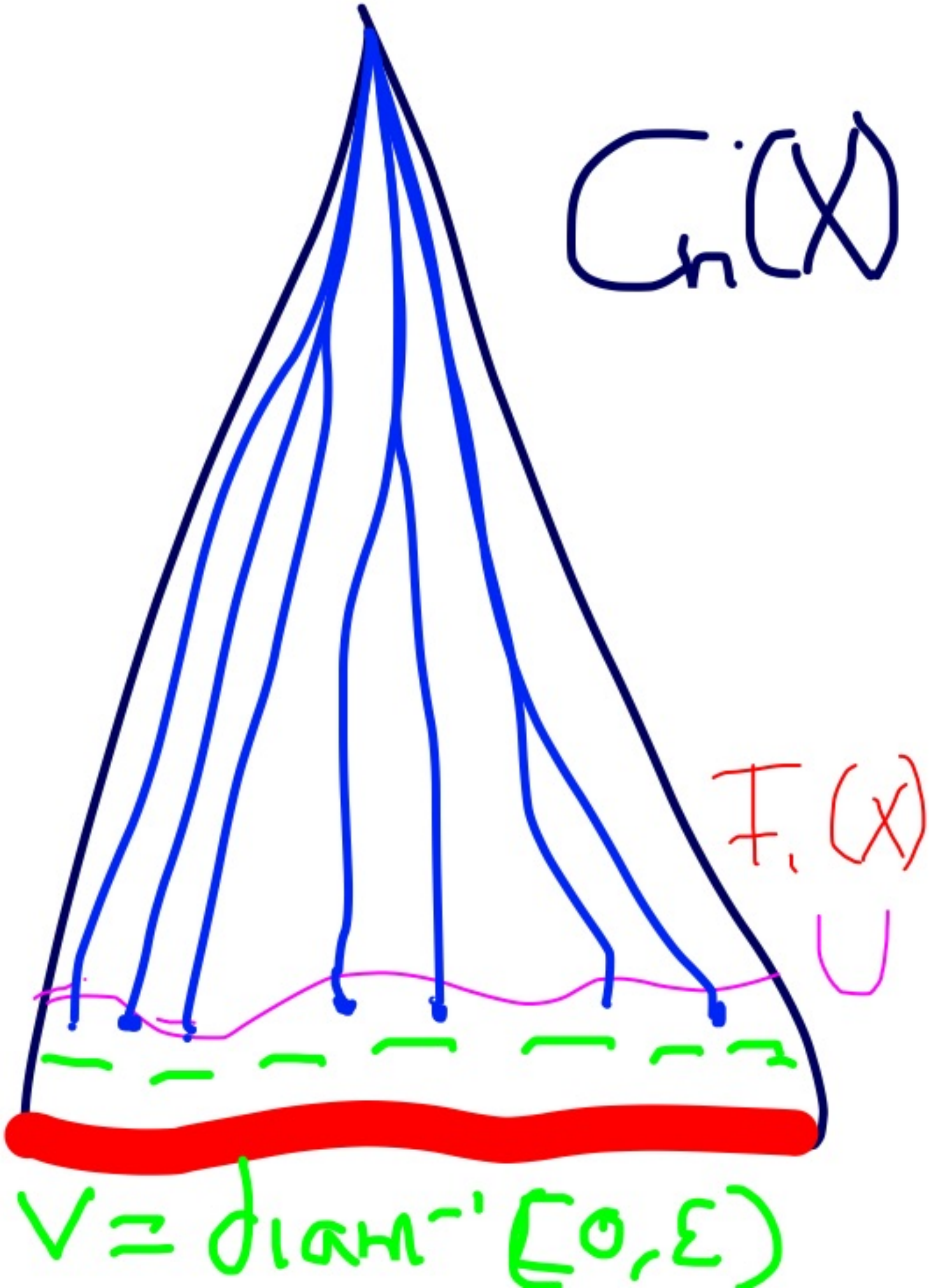


$C_n(X)$
$F_1(X)$
U
$V = \mathrm{diam}^{-1}[0,\varepsilon)$



$C_n(X)$
$F_1(X)$
U
$V = diam^{-1}[0,\varepsilon)$


$C_n(X)$
$F_1(X)$
U
$V = diam^{-1}[0, \varepsilon)$


$C_n(X)$
$F_1(X)$
U
$V = diam^{-1}[0, \varepsilon)$

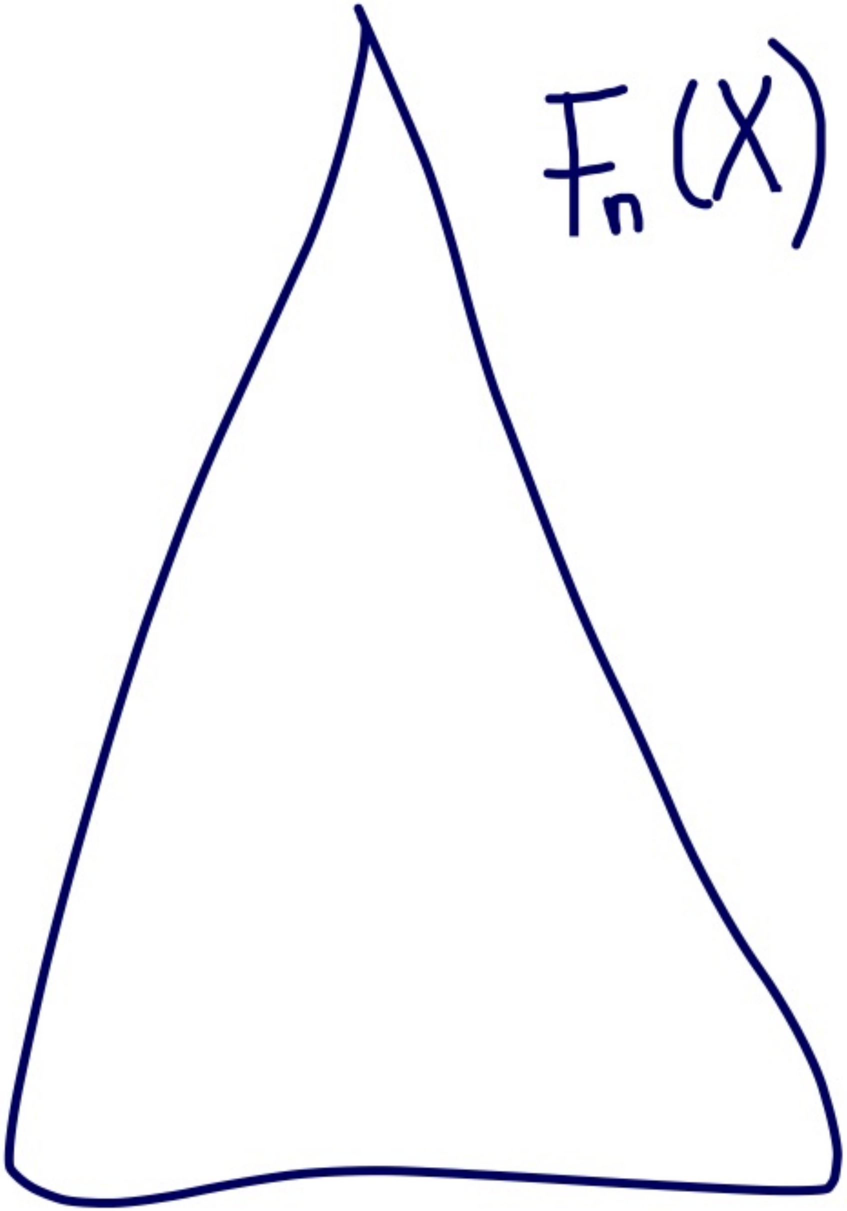
Theorem (VMV and JMM, 2017)

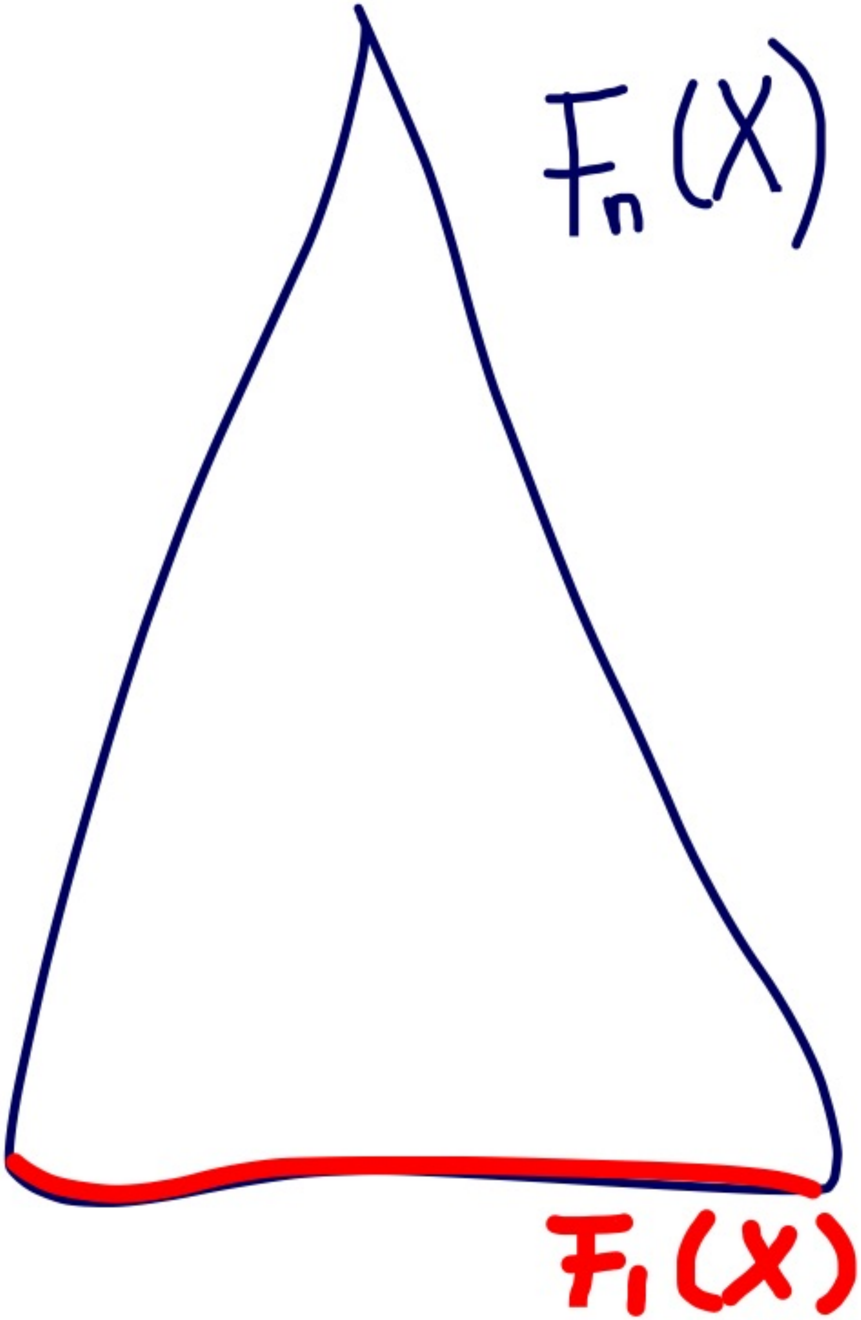
- For every continuum X and each positive integer n, $F_1(X)$ is a colocally connected subcontinuum in 2^X

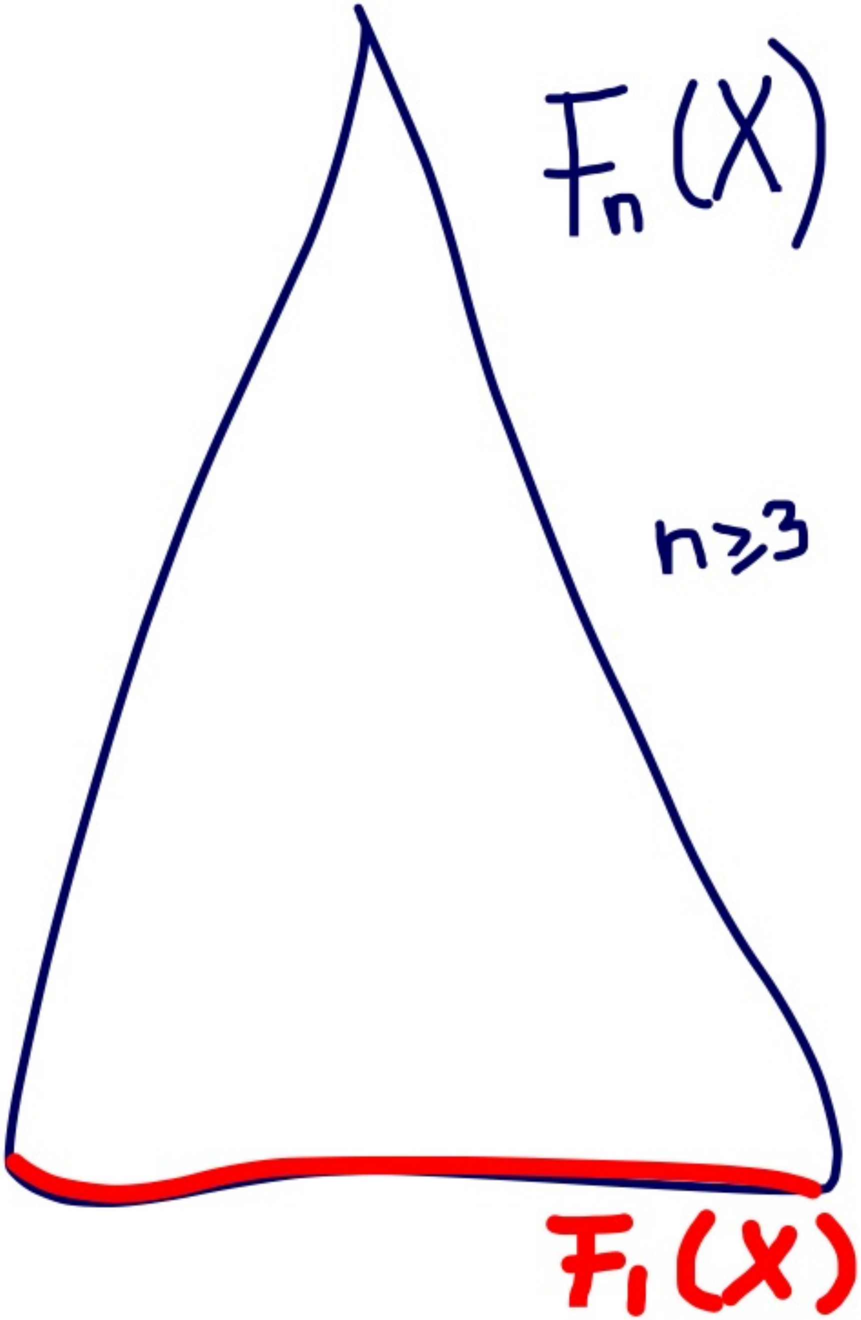
4 Main Results in Symmetric Products.

Theorem (VMV and JMM, 2017)

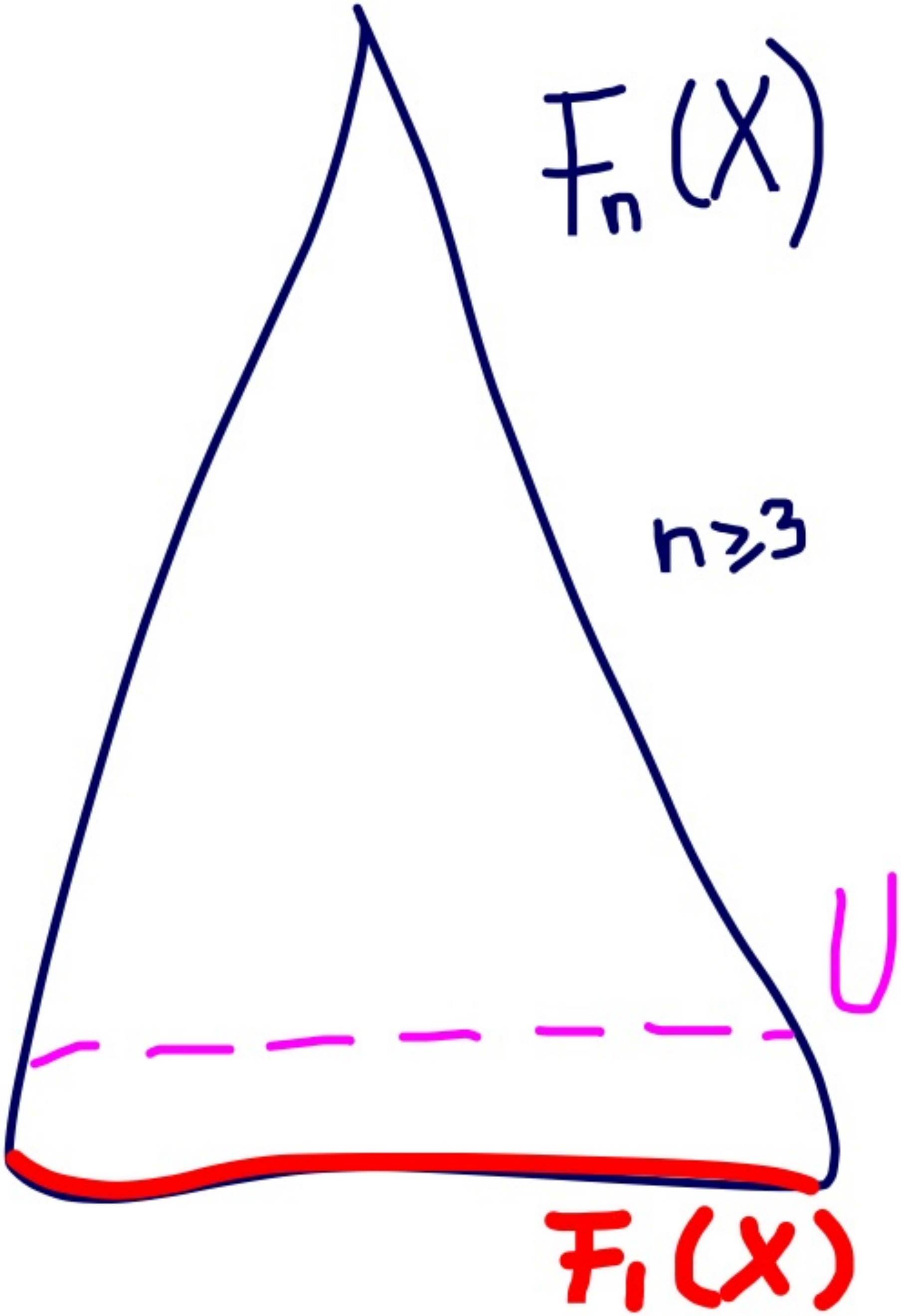
- For every continuum X and each positive integer $n \geq 3$, $F_1(X)$ is a colocally connected set in $F_n(X)$


$F_n(X)$

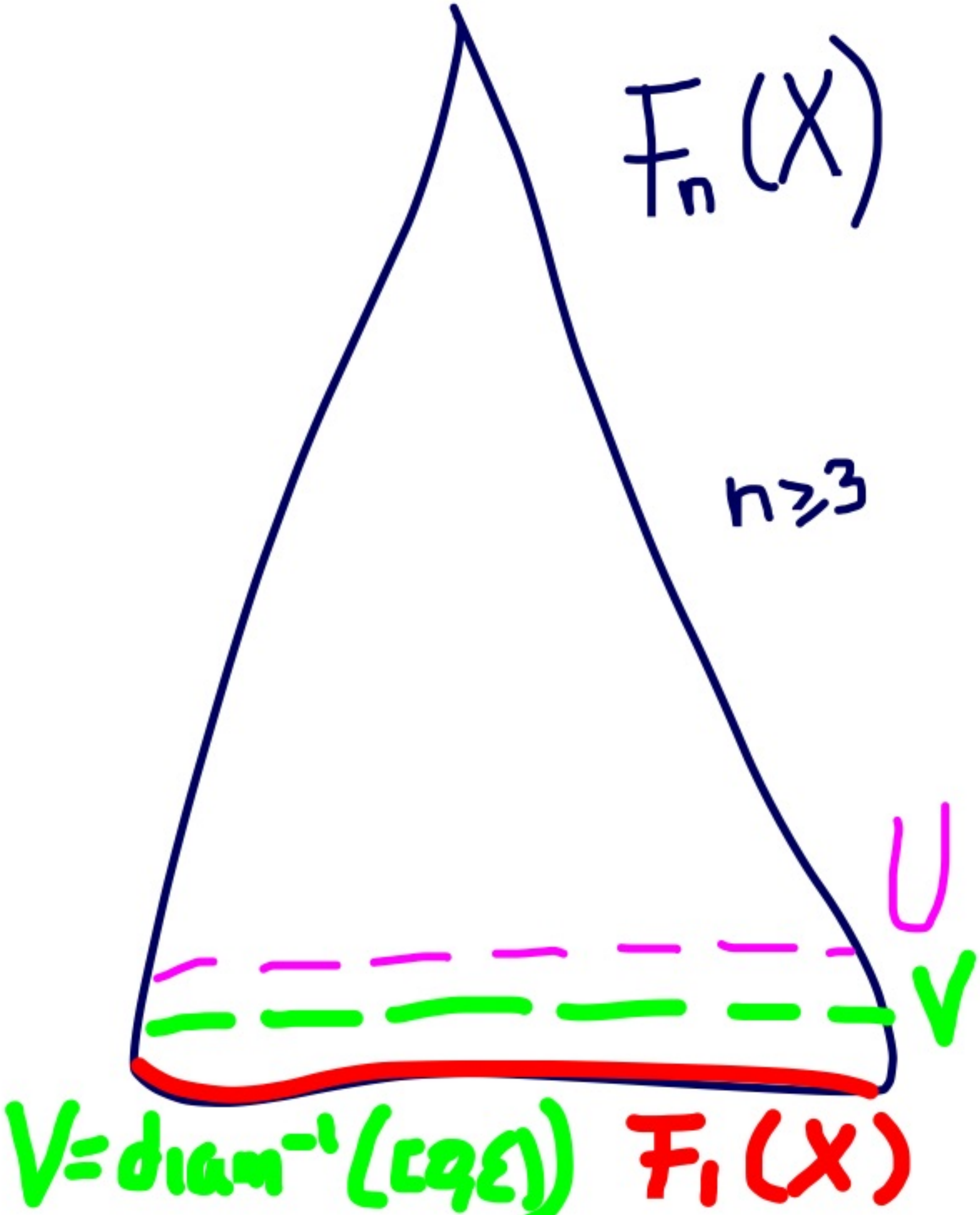

$F_n(X)$
$F_1(X)$



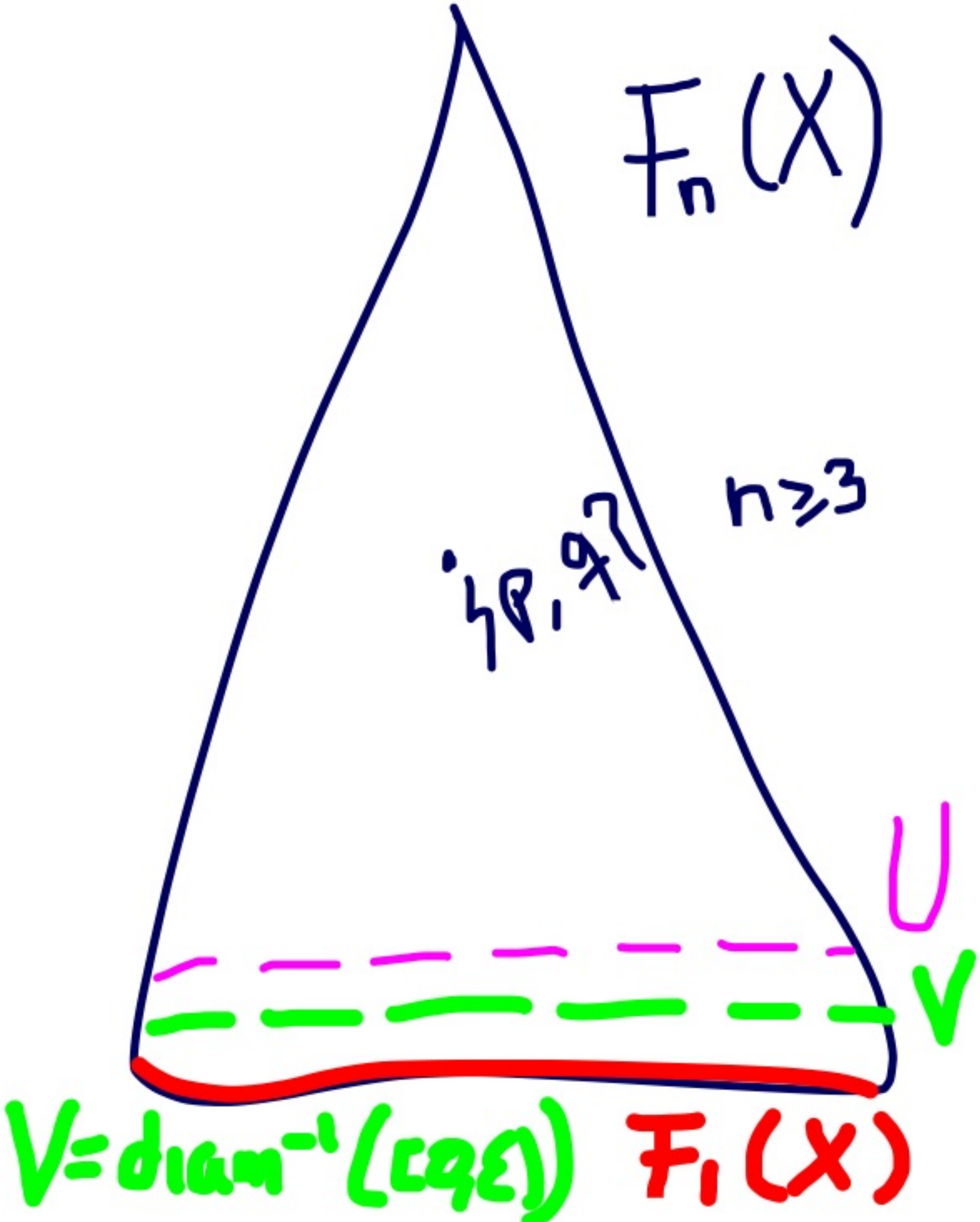
$F_n(X)$
$n \geq 3$
$F_1(X)$



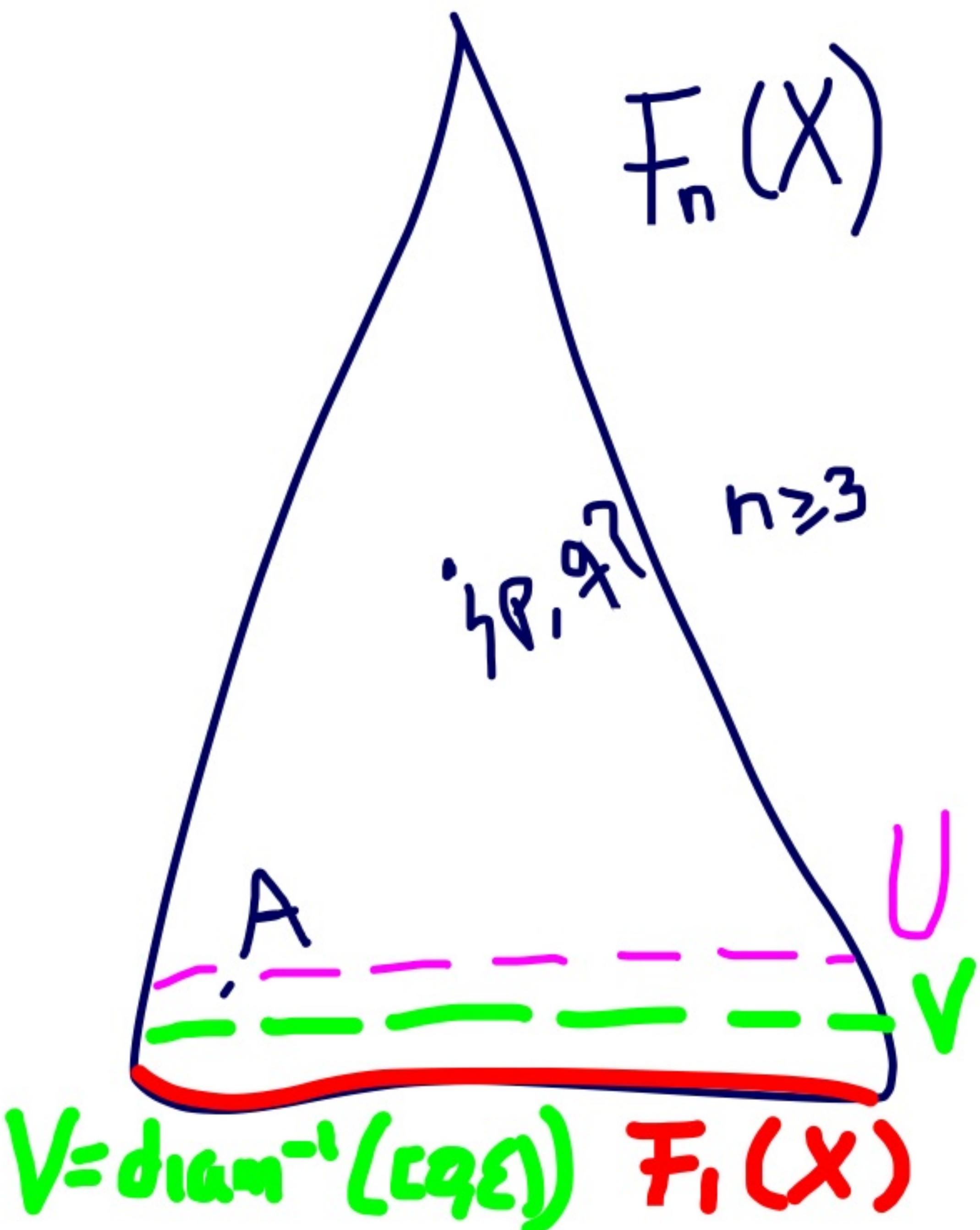
$F_n(X)$
$n \geq 3$
U
$F_1(X)$



$F_n(X)$
$n \geq 3$
U
V
$V = diam^{-1}([0,\varepsilon))$
$F_1(X)$



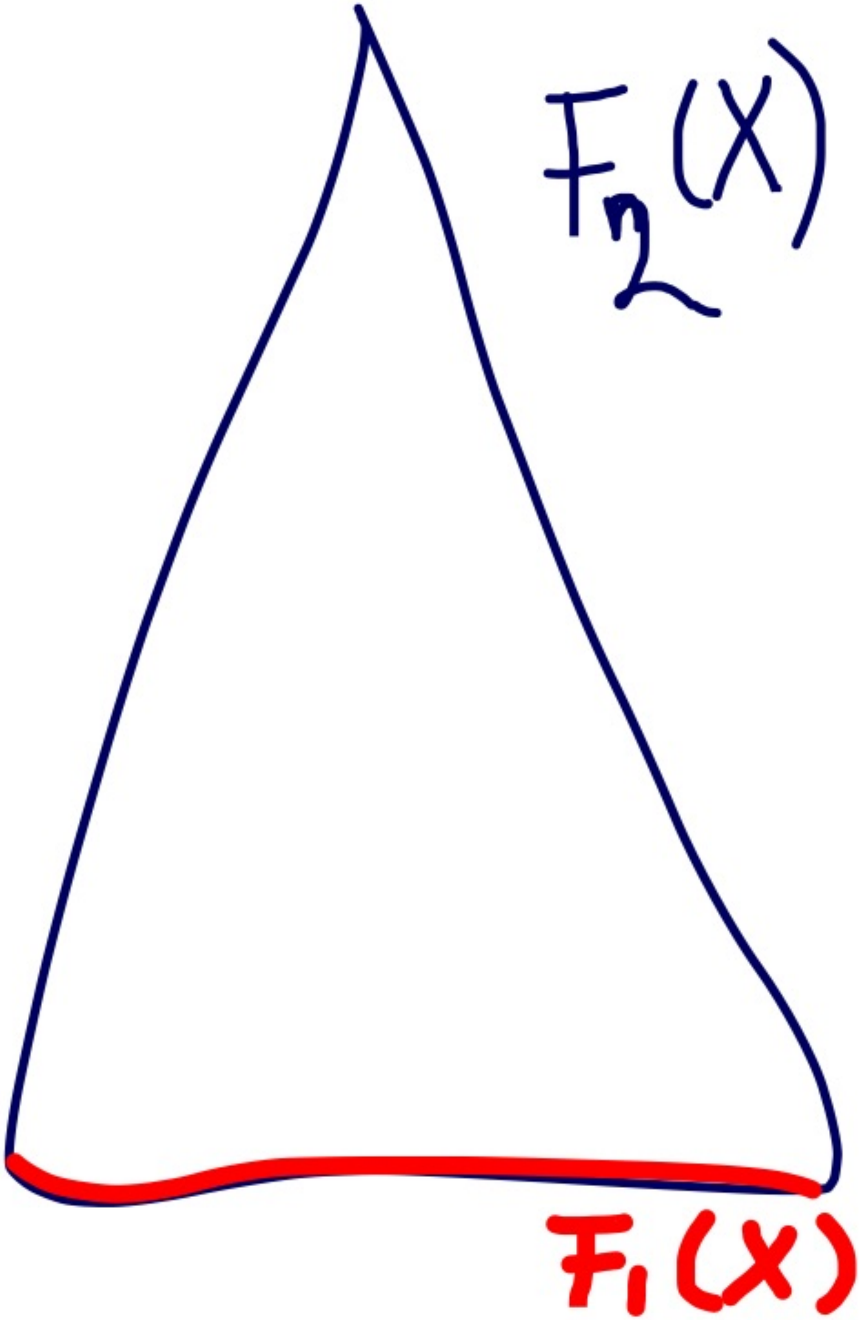
Fn(X)
n≥3
{p,q}
U
V
V=diam⁻¹([0,ε))
F1(X)



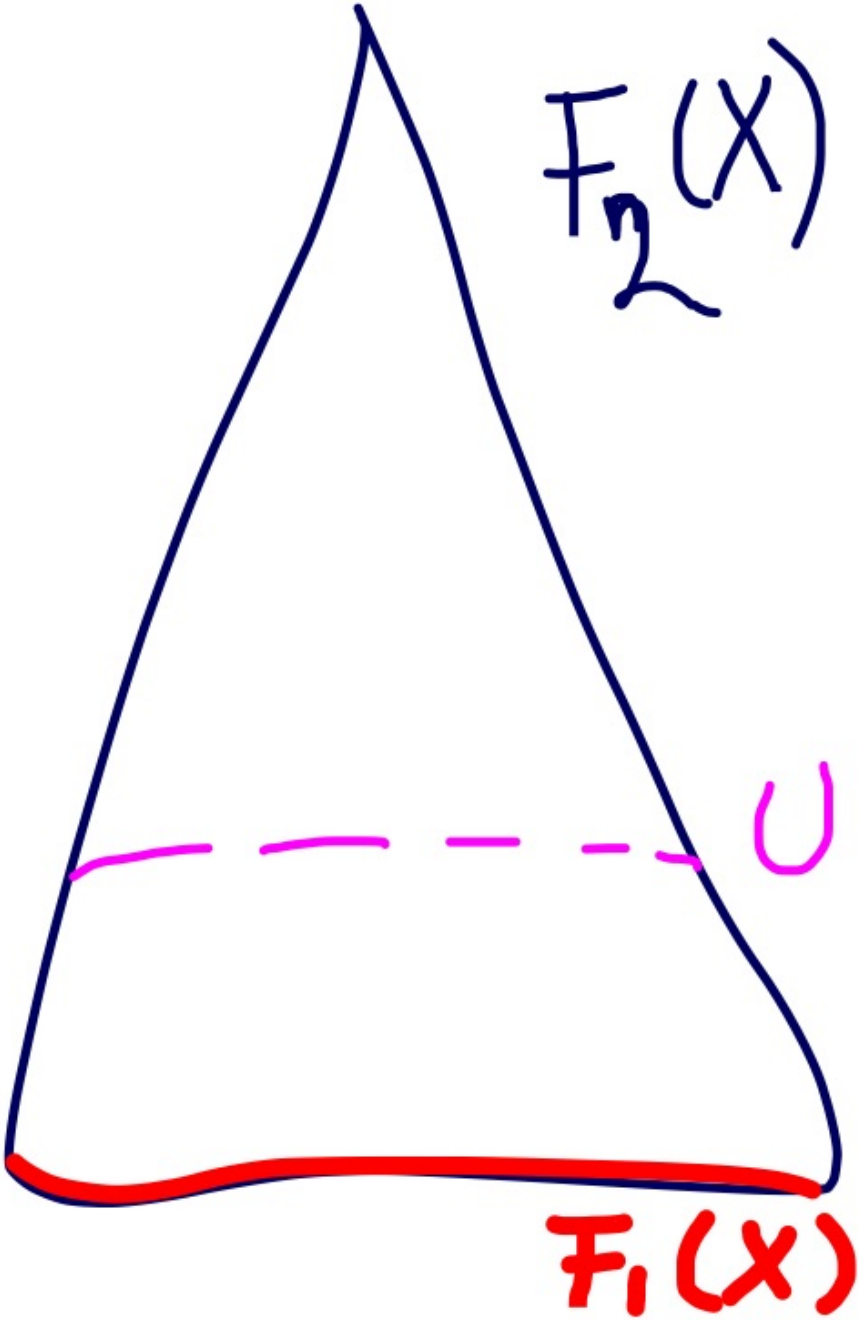
$F_n(X)$
$n \geq 3$
$\{p, q\}$
A
U
V
$V = diam^{-1}([0, \varepsilon))$
$F_1(X)$

Theorem (VMV and JMM, 2017)

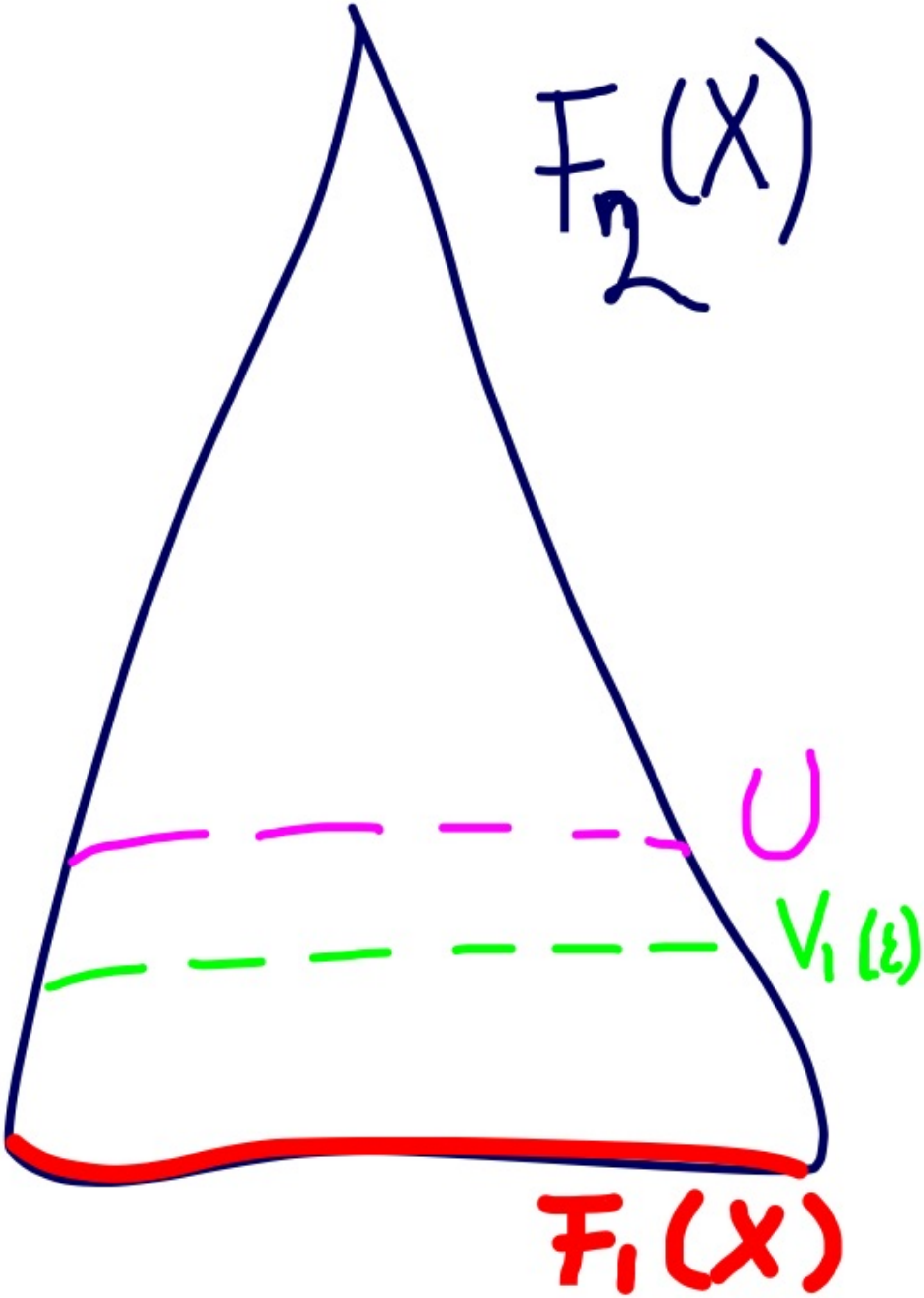
- For every locally connected continuum X and each integer n, n = 2, $F_1(X)$ is a colocally connected set in $F_n(X)$



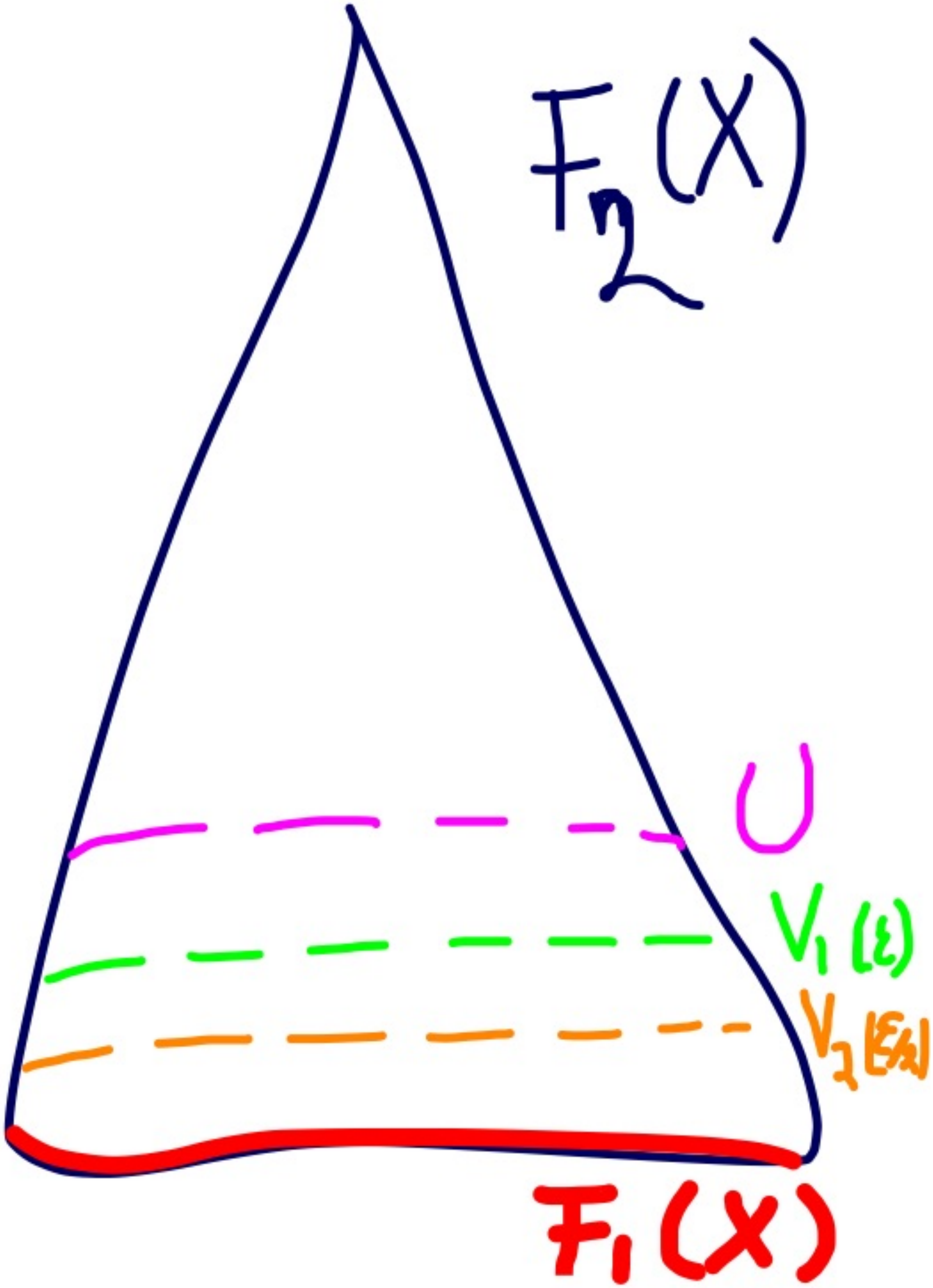
F2(X)
F1(X)

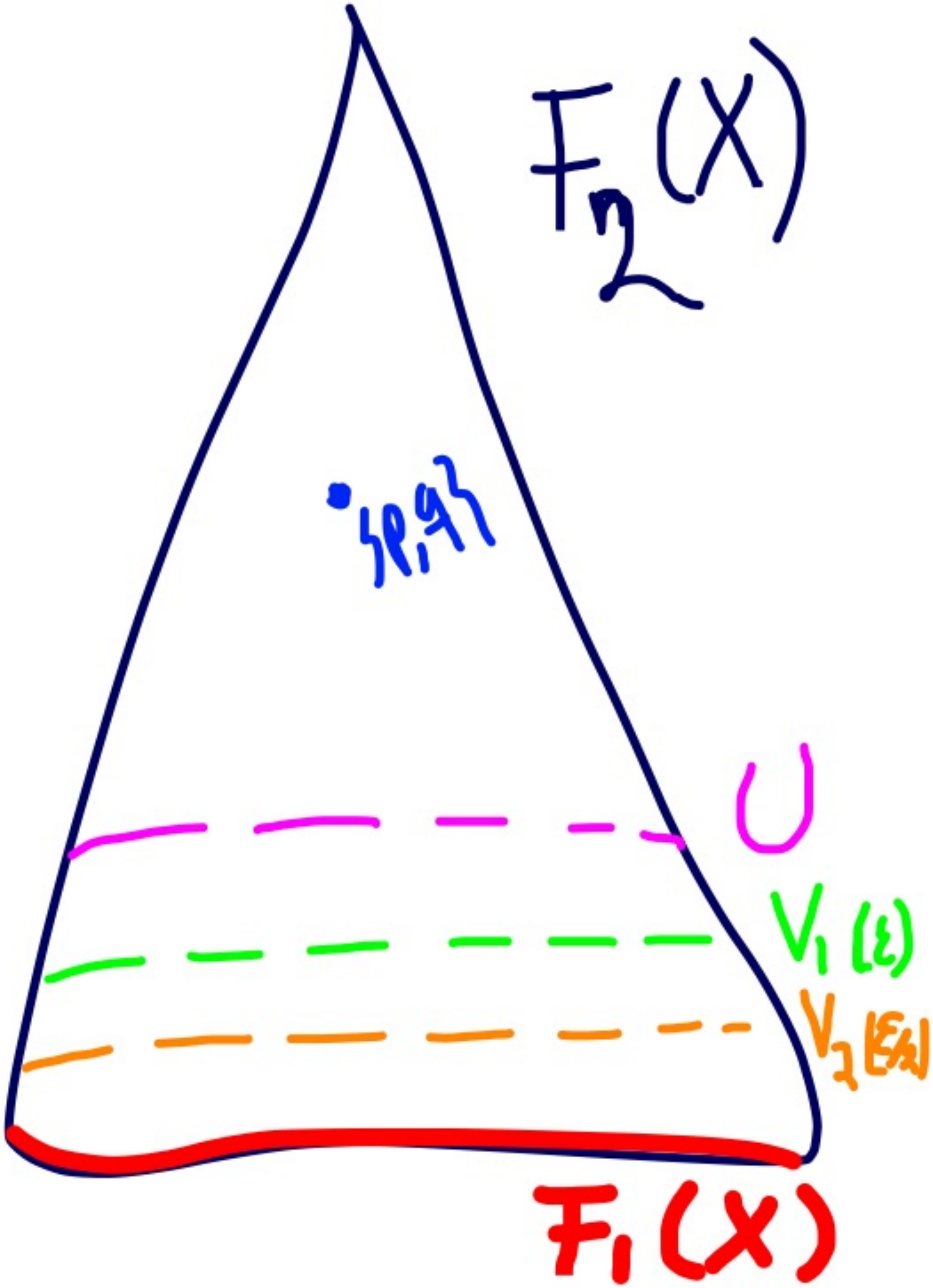


$F_2(X)$
U
$F_1(X)$

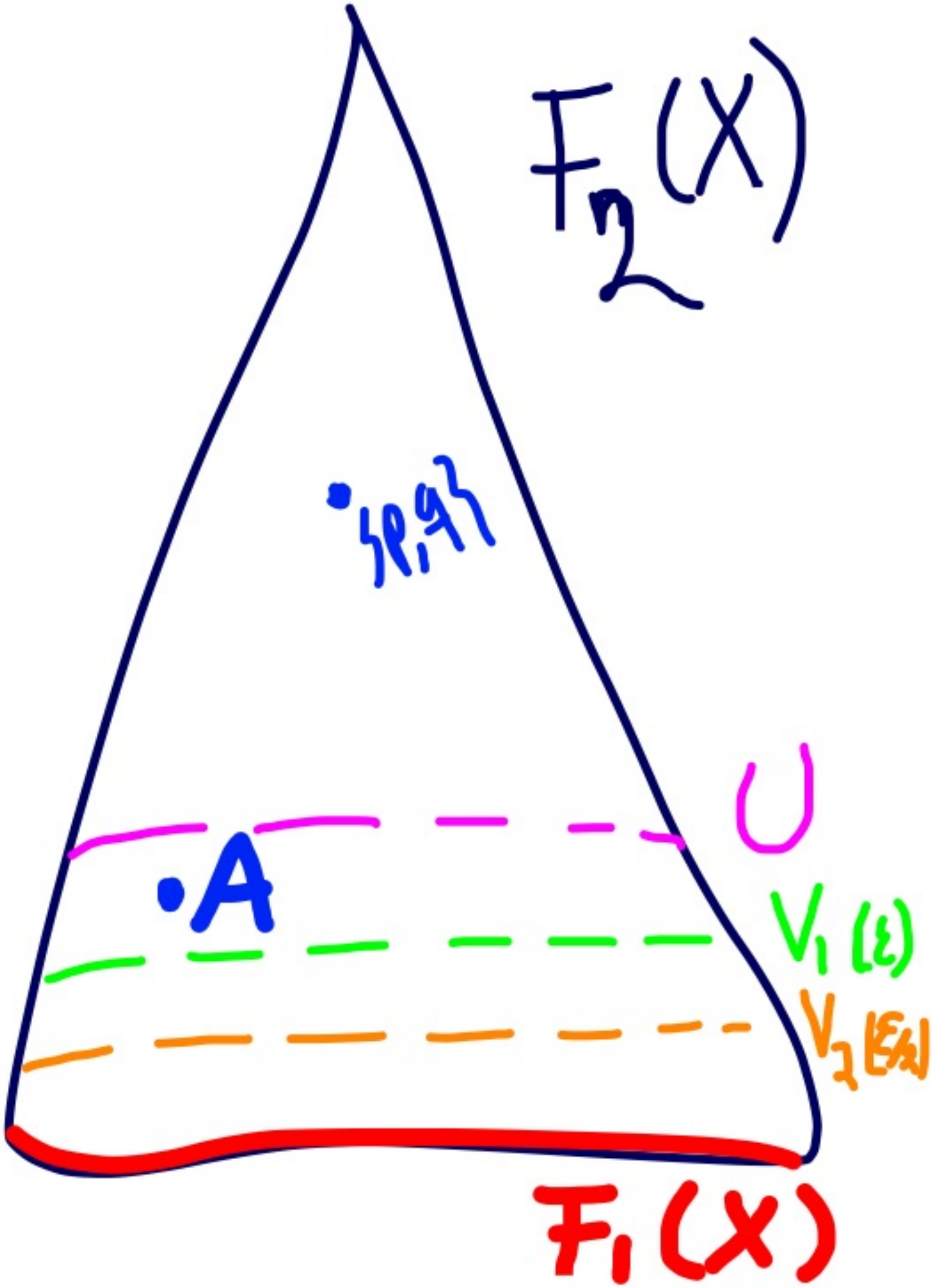


F2(X)
U
V1(ε)
F1(X)

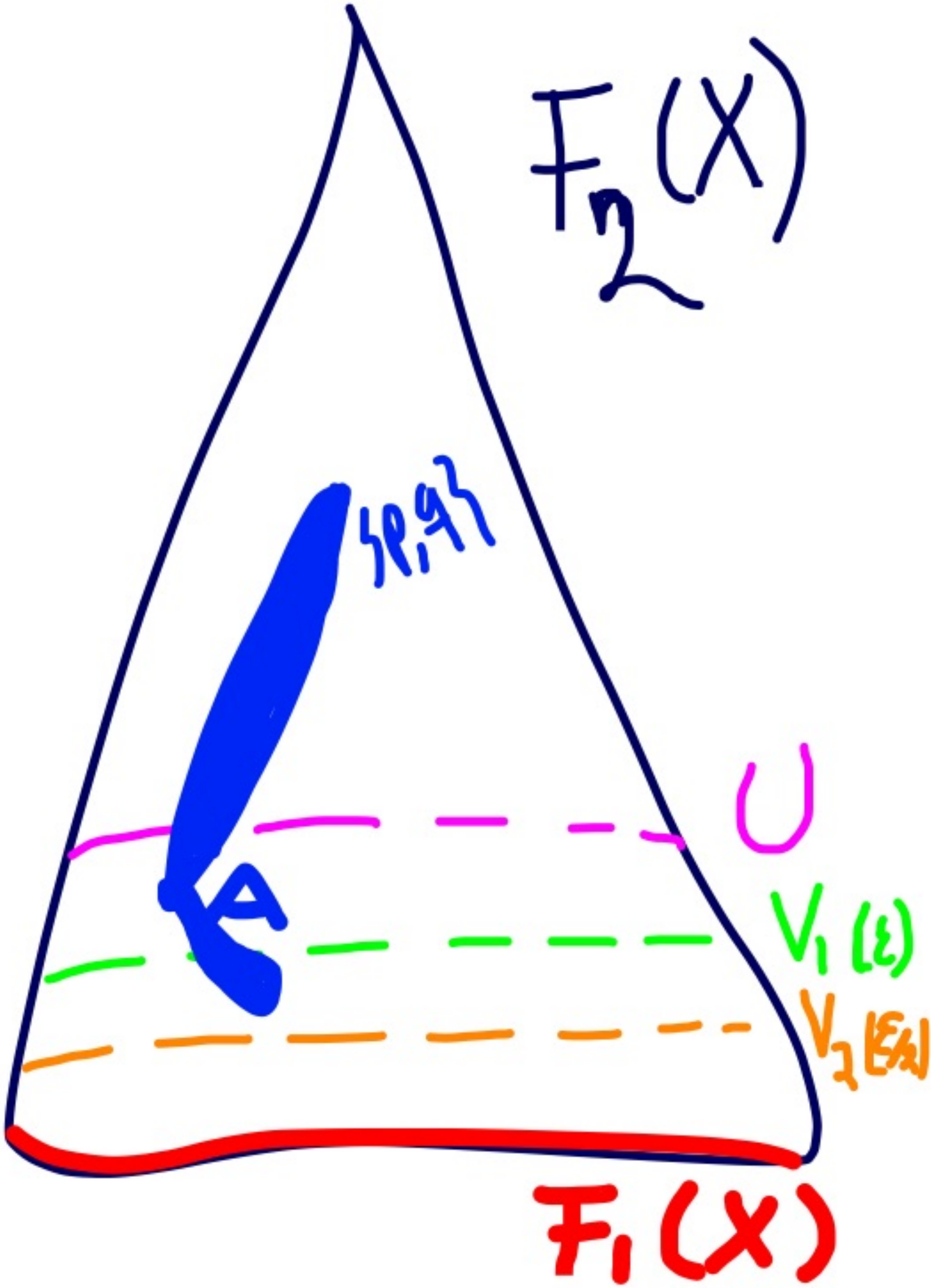

F2(X)
U
V1(ε)
V2
F1(X)



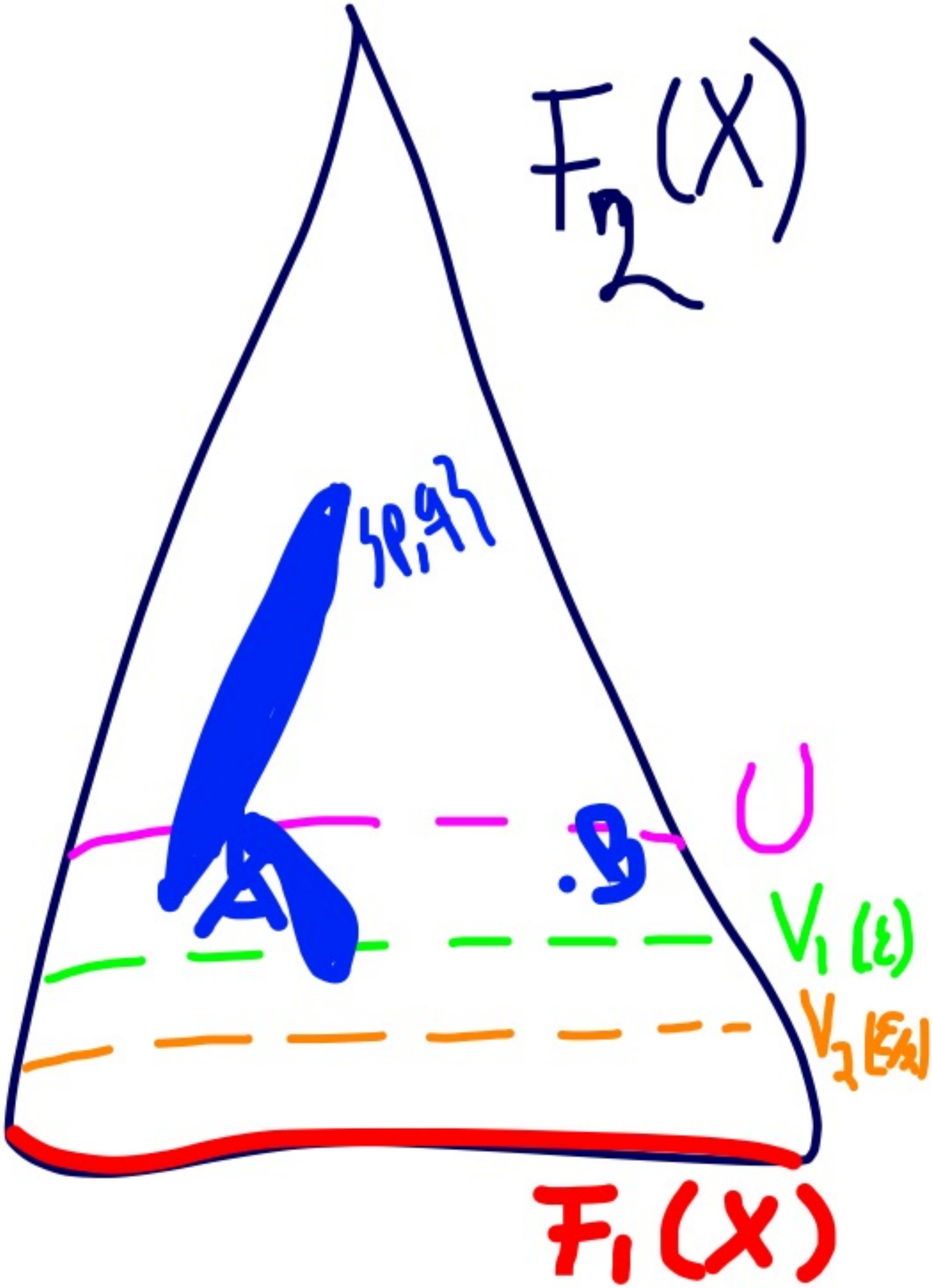
$F_2(X)$
$\{p,q\}$
U
$V_1(\varepsilon)$
$V_2(\varepsilon_2)$
$F_1(X)$



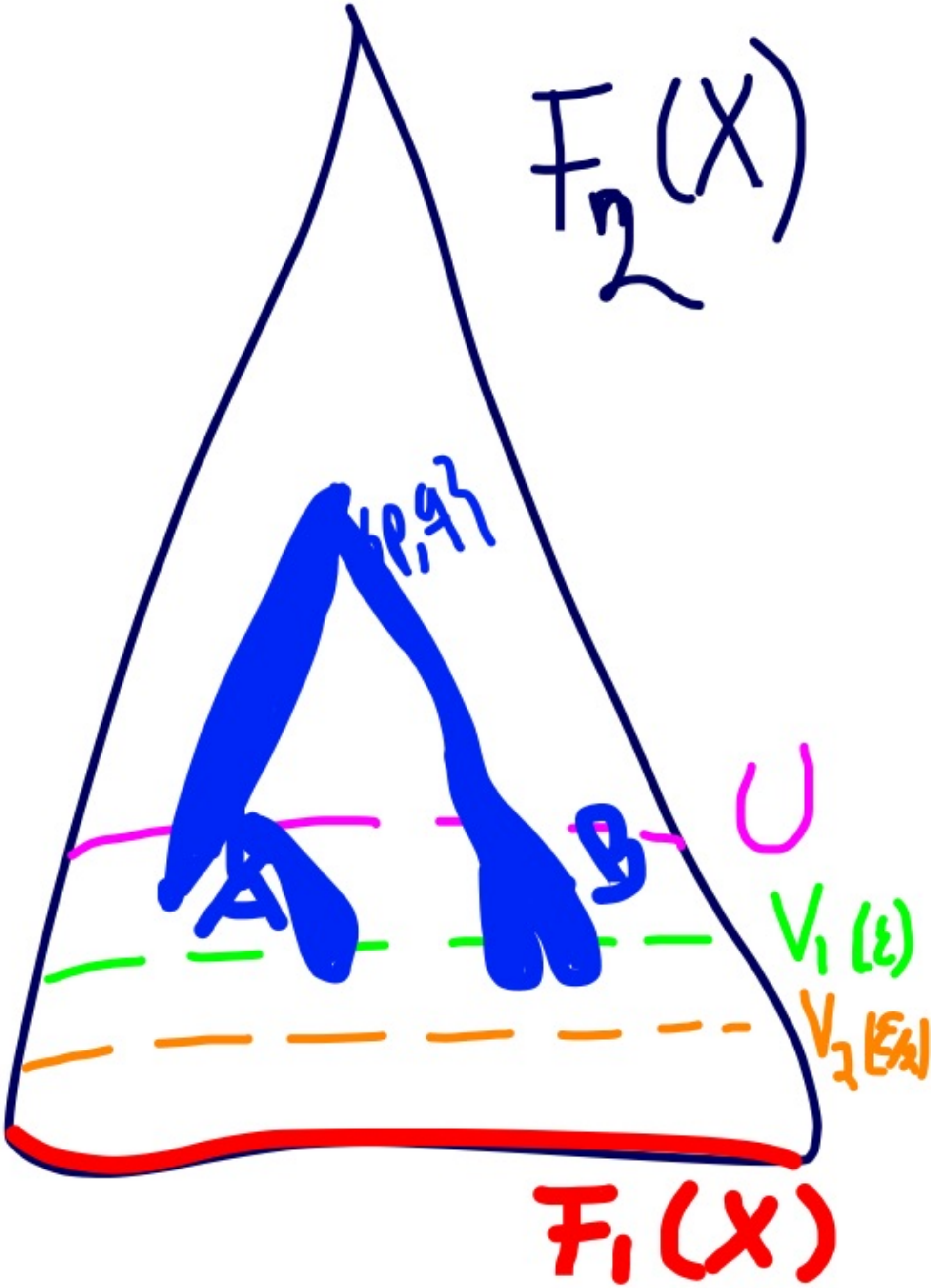
F2(X)
{p,q}
A
U
V1(ε)
V2
F1(X)



F_2(X)
{p,q}
A
U
V_1(ε)
V_2(ε/2)
F_1(X)



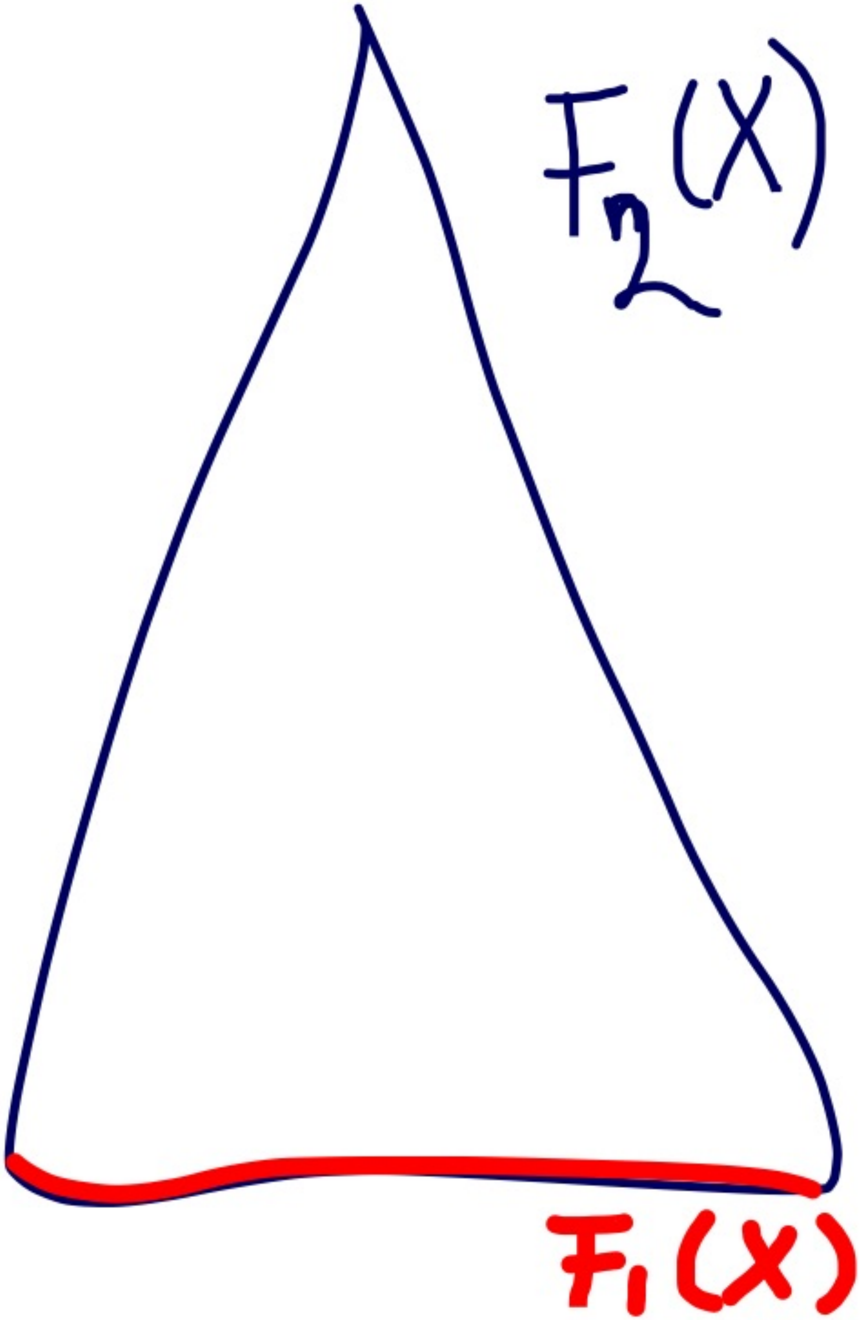
$F_2(X)$
{p,q}
A
B
U
$V_1(\varepsilon)$
$V_2(\varepsilon_A)$
$F_1(X)$

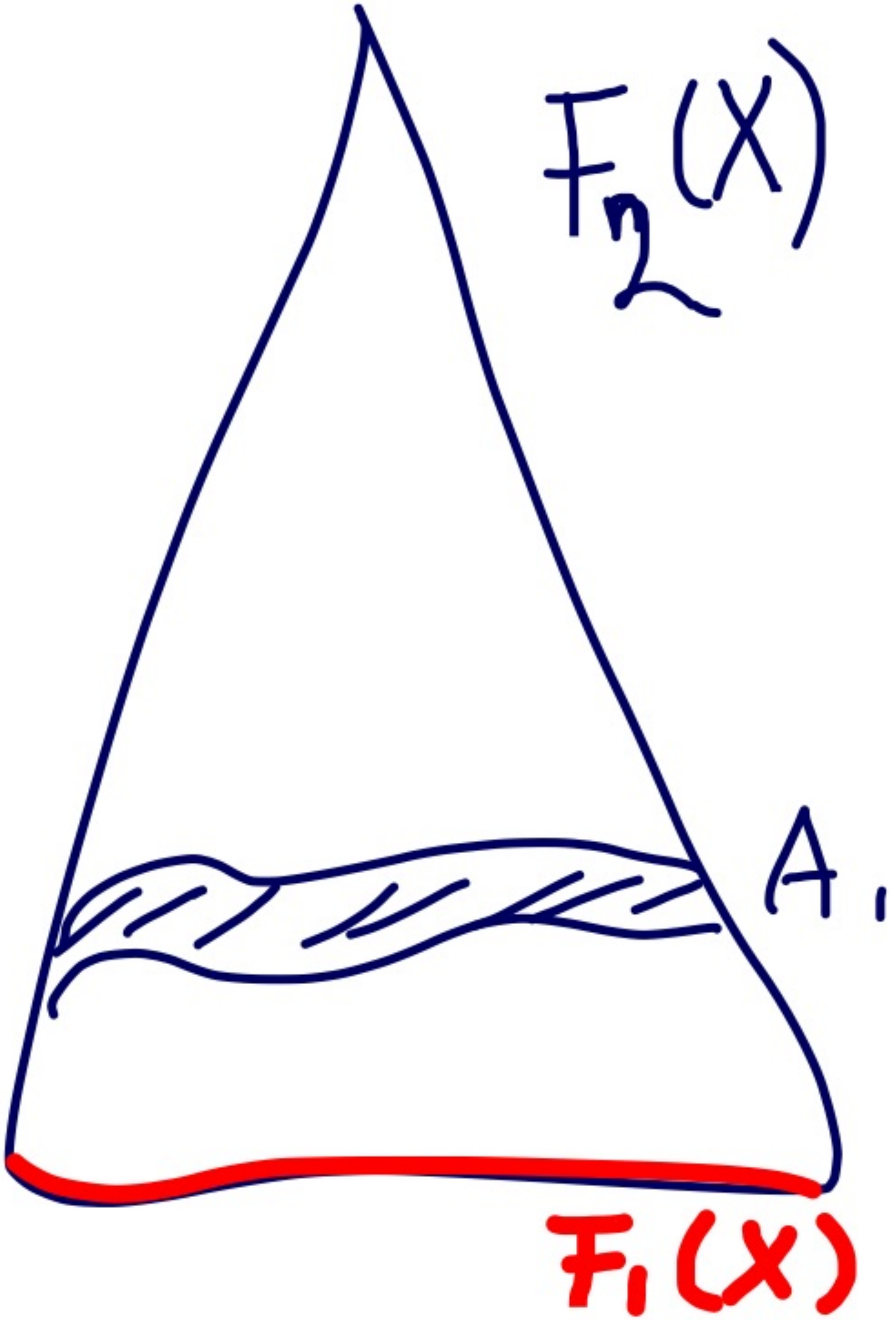


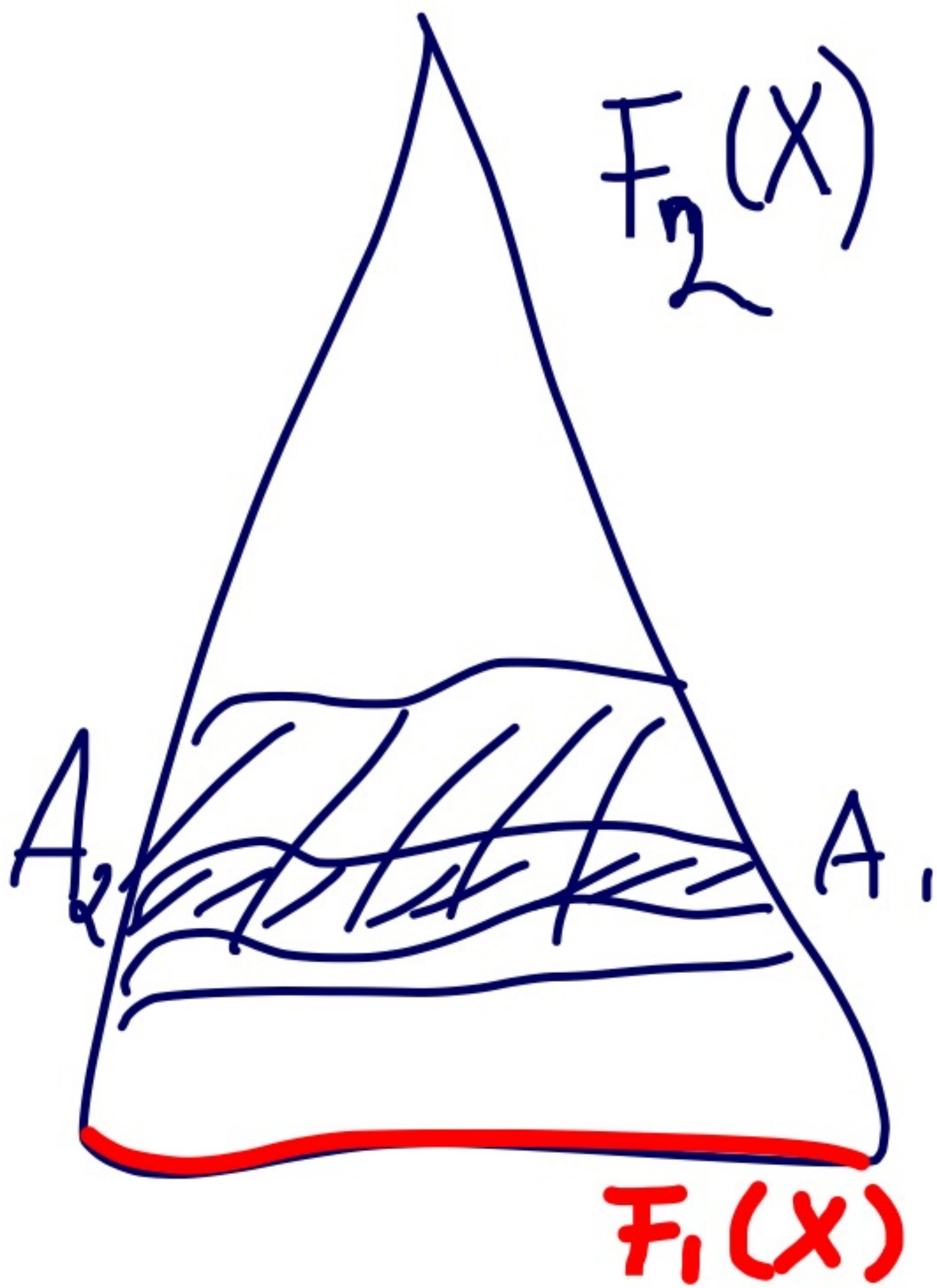
F2(X)
{p,q}
U
A
B
V1(ε)
V2(ε/2)
F1(X)

Theorem (VMV and JMM, 2017)

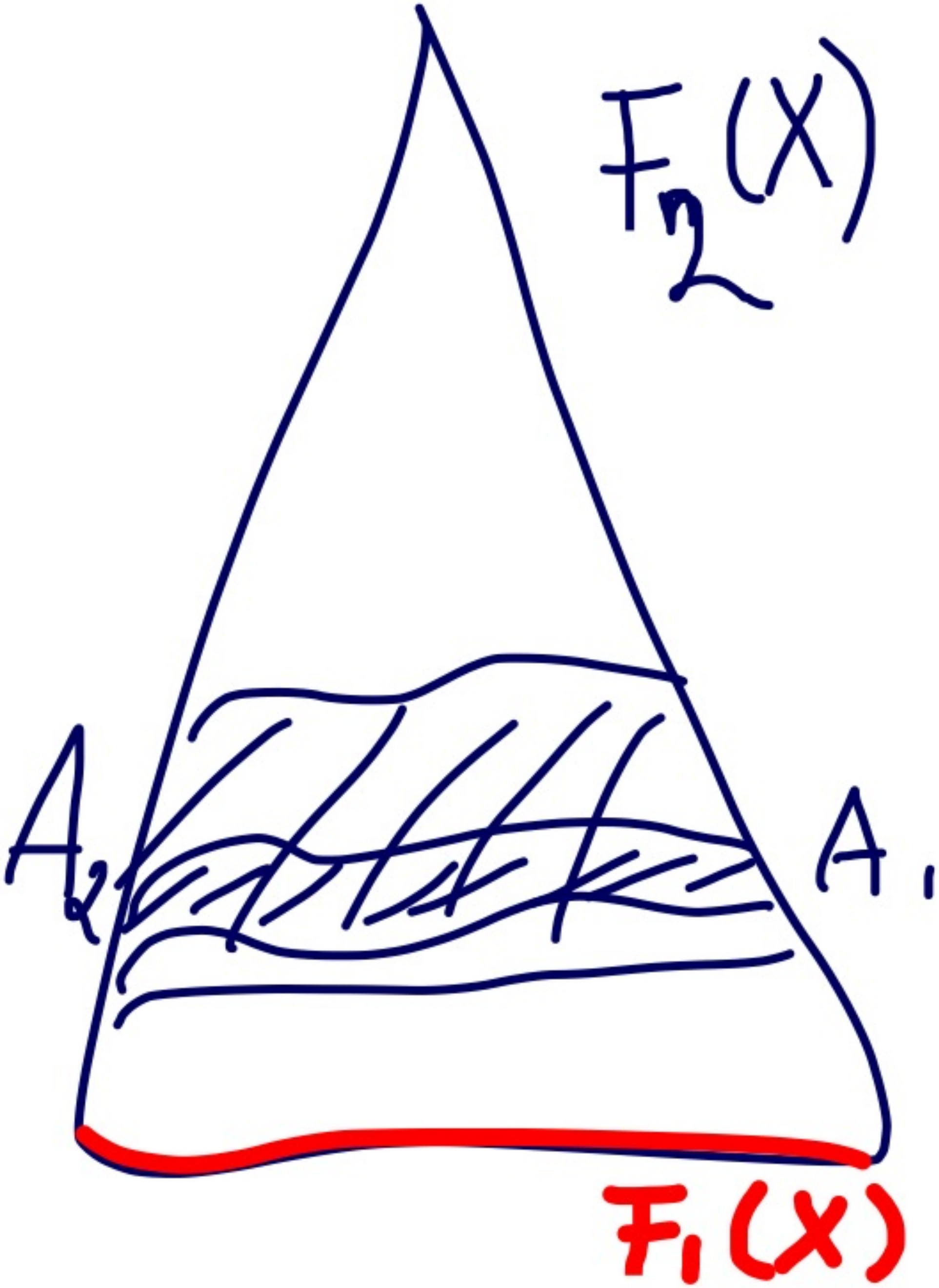
- Let X be a continuum and $n = 2$. Then $F_1(X)$ is a nonblock continuum in $F_n(X)$.


$F_2(X)$
$F_1(X)$

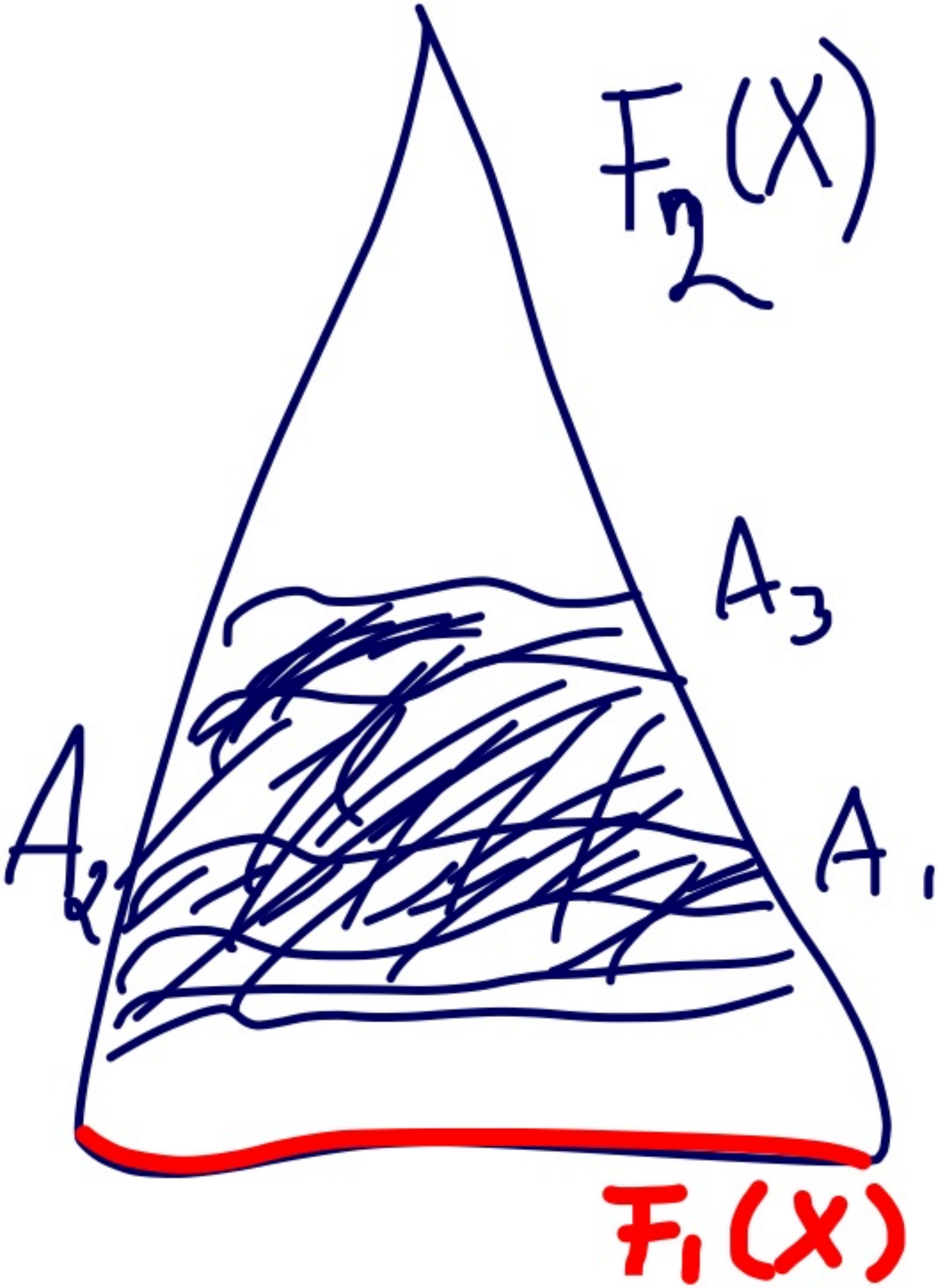

F2(X)
A
F1(X)



$F_2(X)$
A_2
A_1
$F_1(X)$



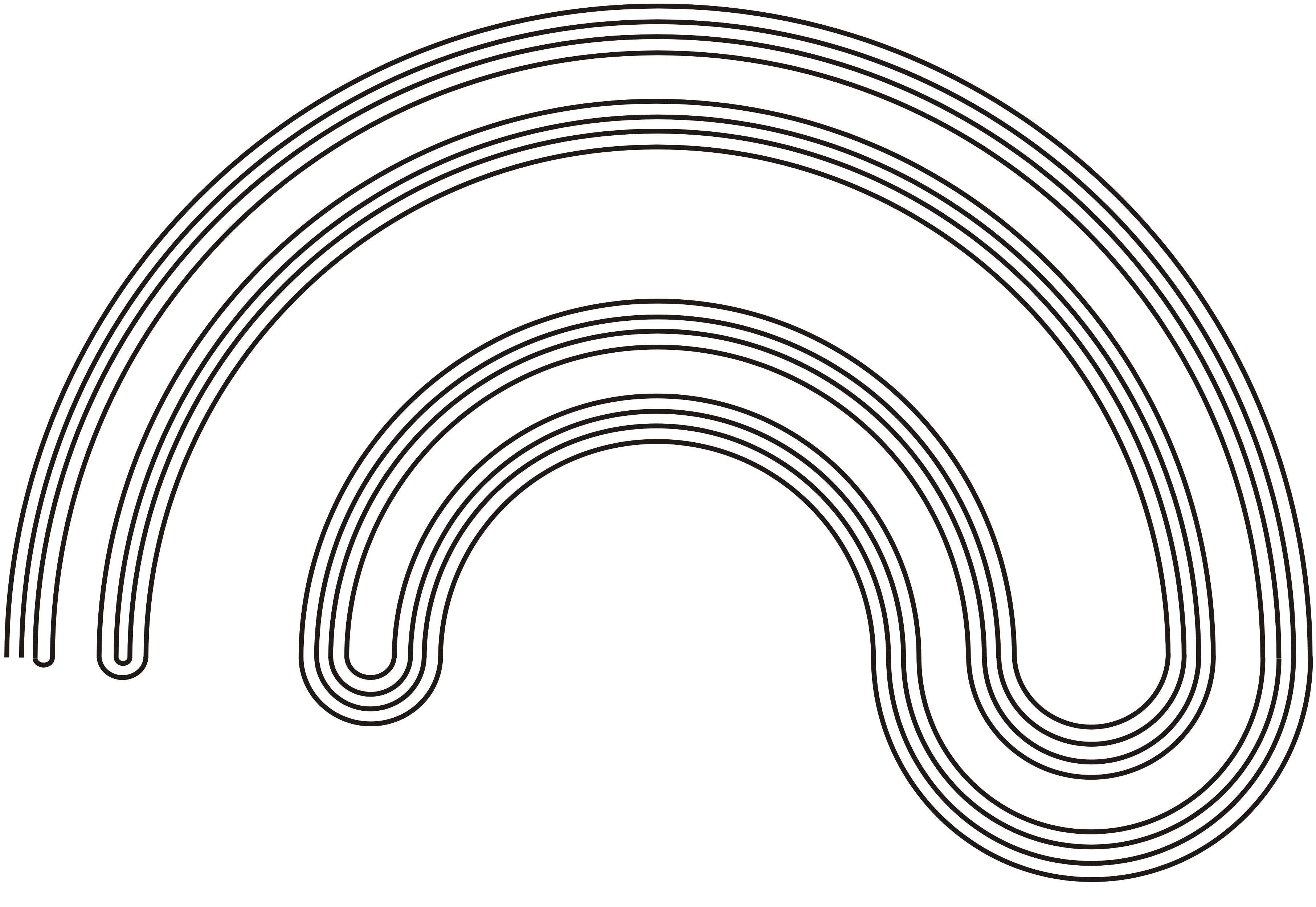
$F_2(X)$
A_2
A_1
$F_1(X)$



$F_2(X)$
A_3
A_2
A_1
$F_1(X)$

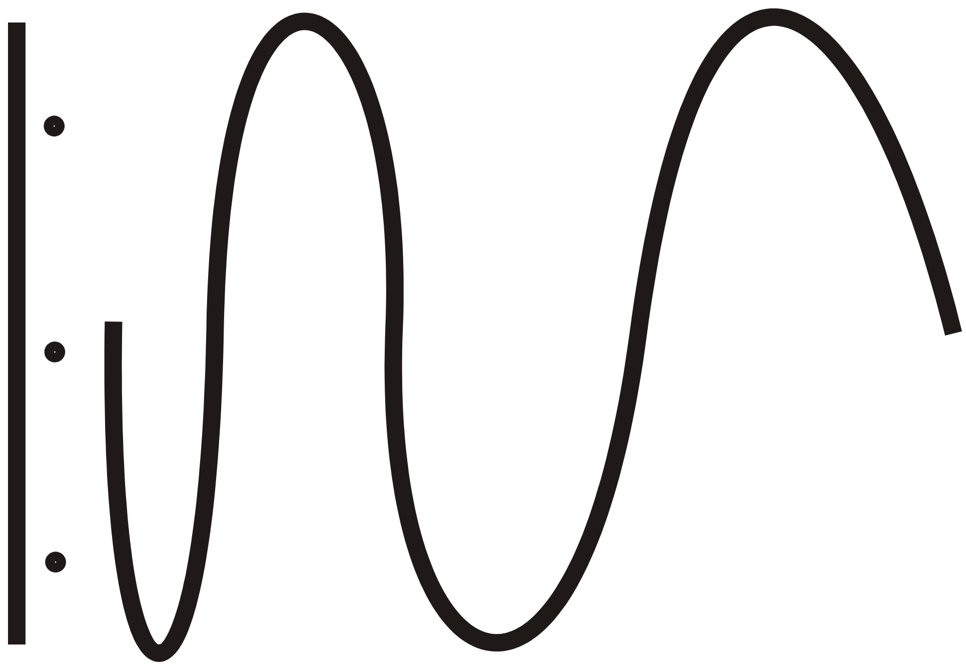
Theorem (VMV and JMM, 2017)

- The Knaster continuum K satisfies that $F_1(K)$ is not colocally connected in $F_2(X)$

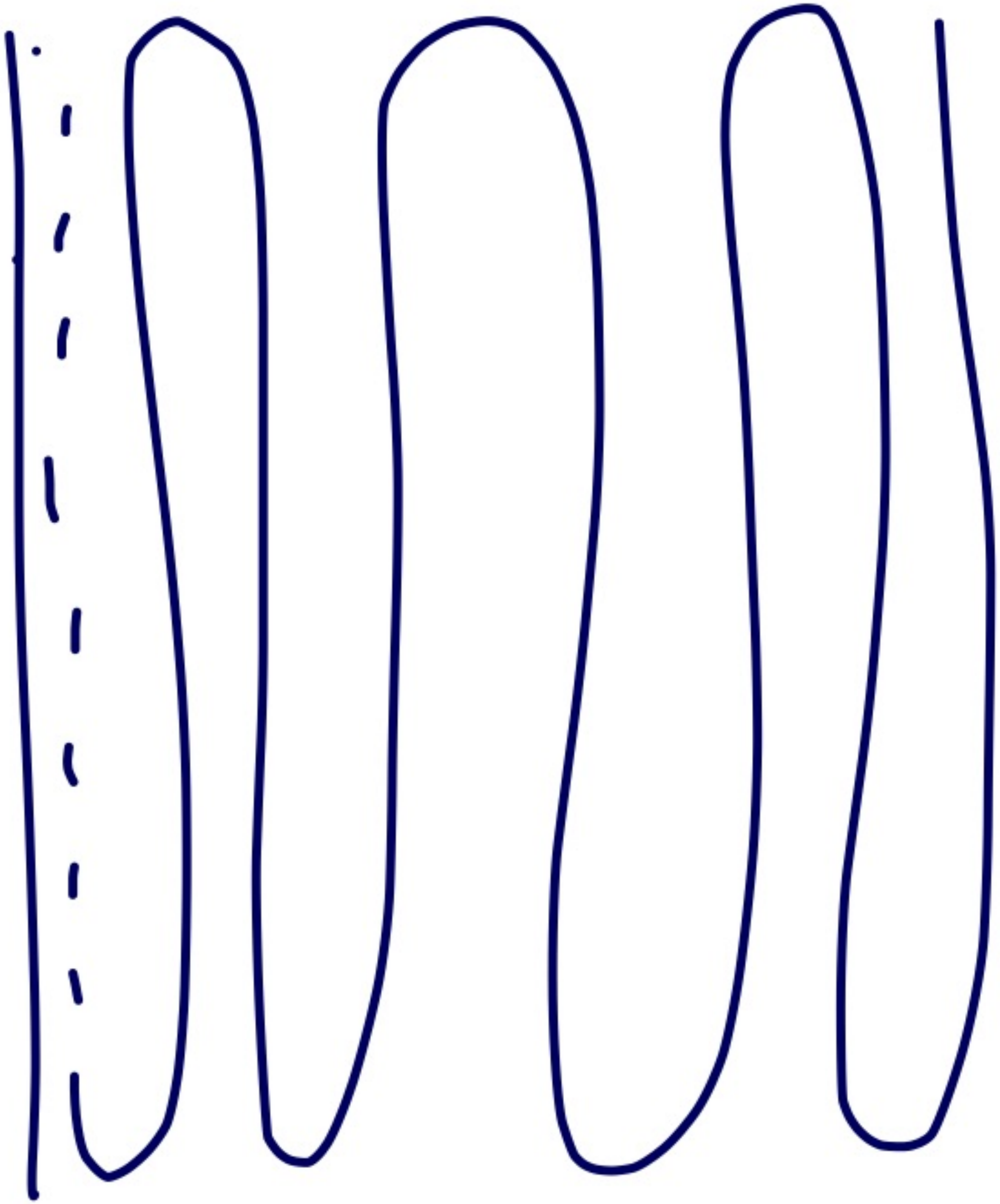


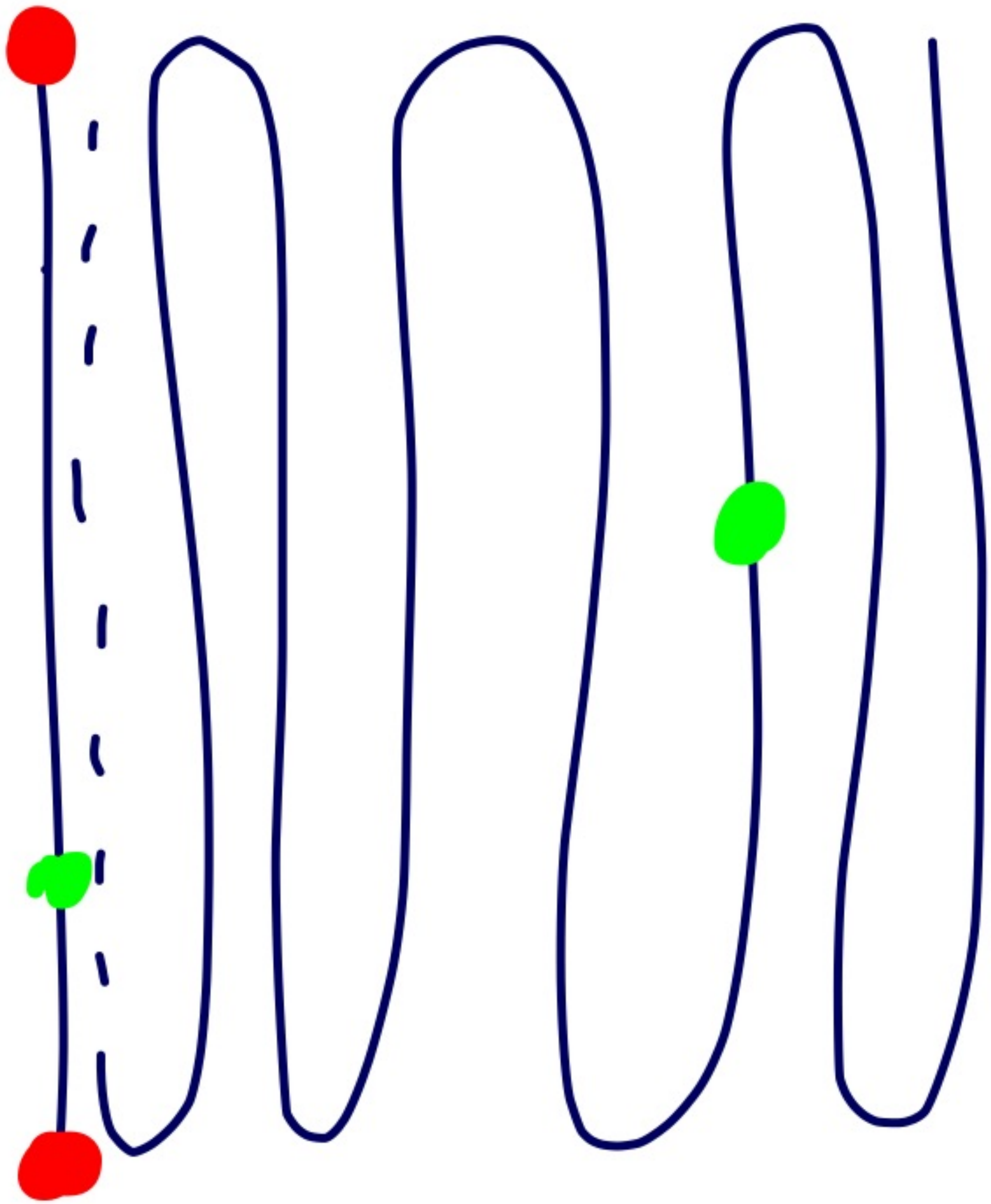
Theorem (VMV and JMM, 2017)

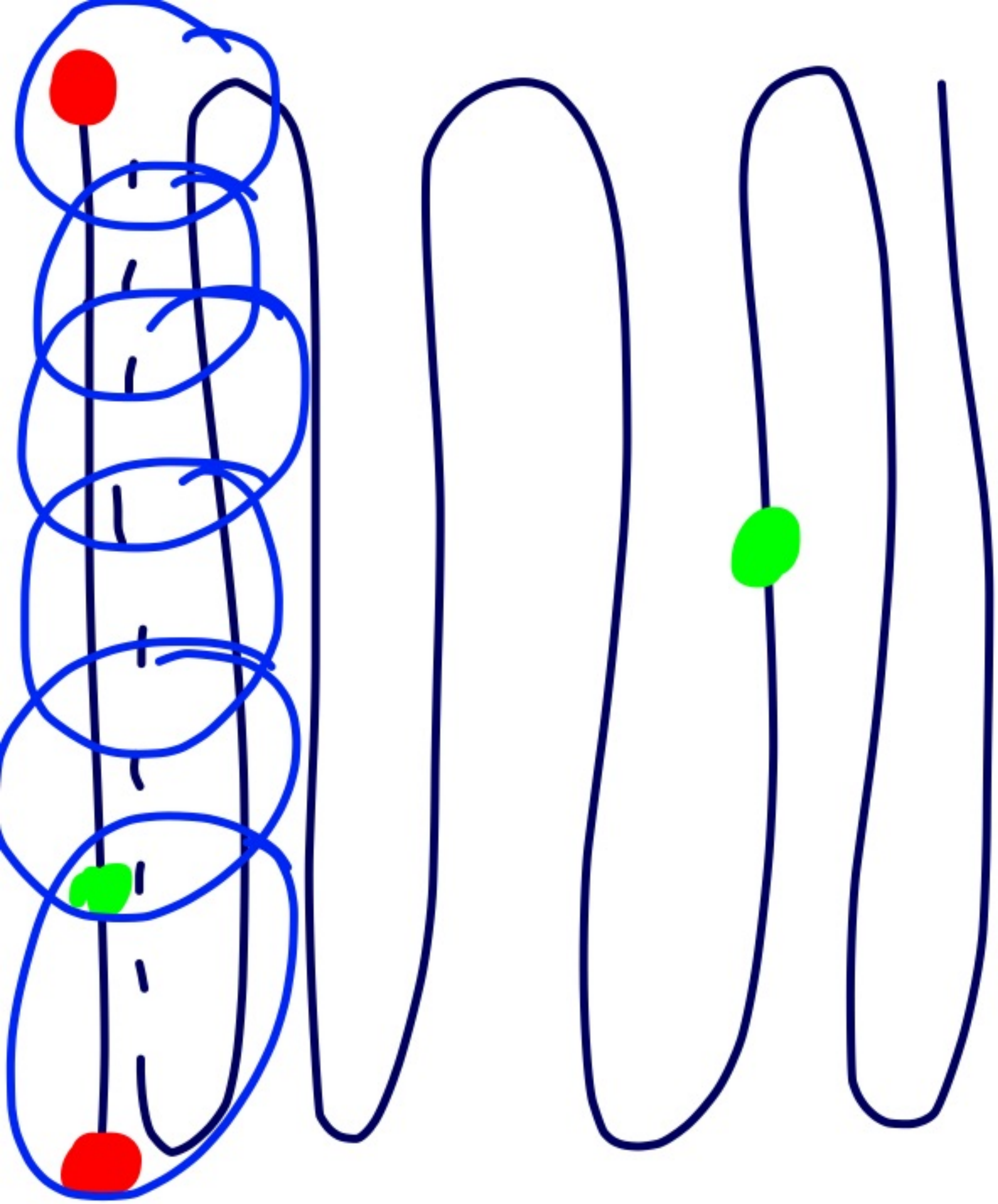
- Let X be the sin 1/x continuum, then $F_1(X)$ is a weak cut continuum in $F_2(X)$.



Sen$(1/x)$-continuum







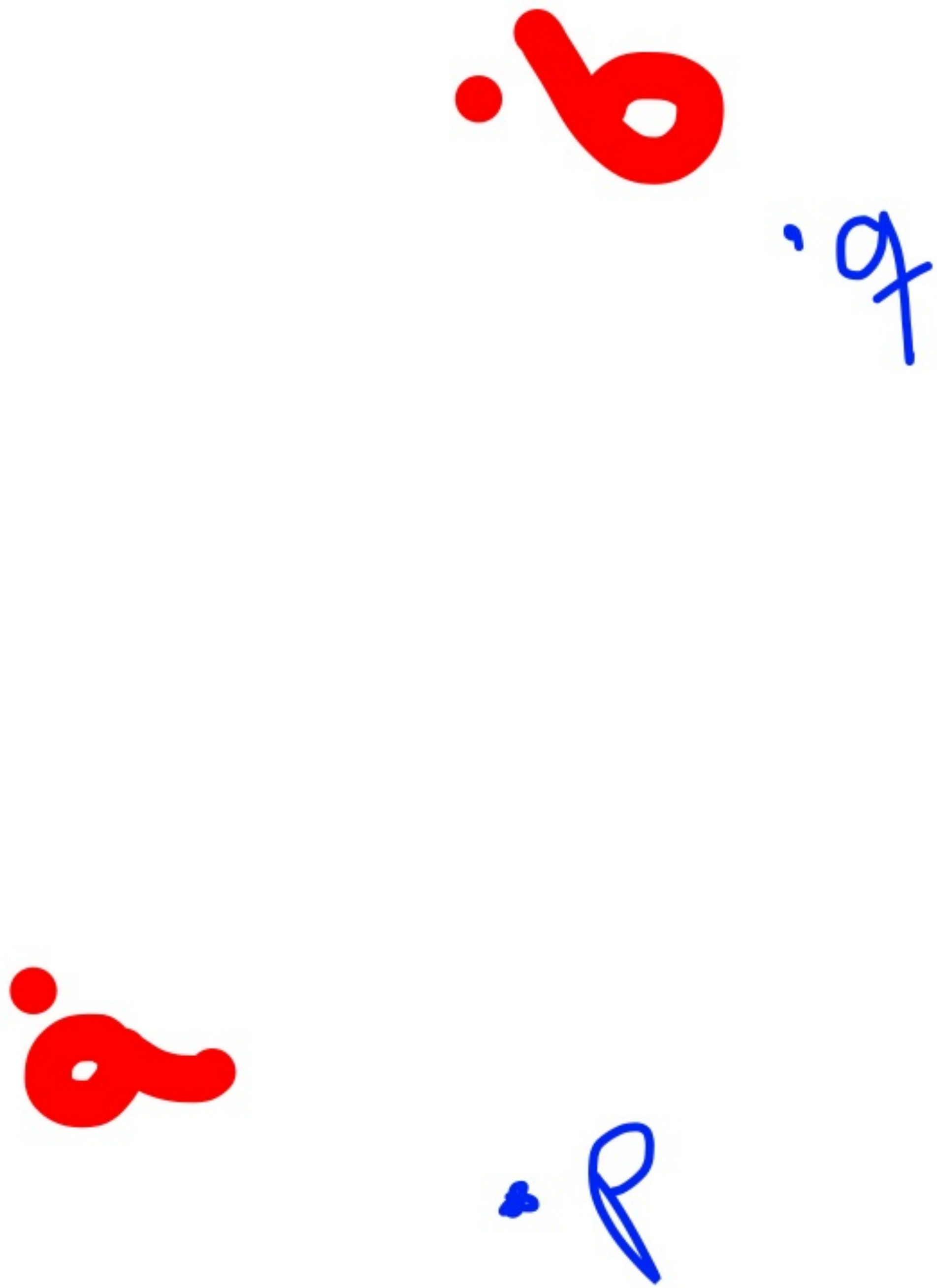
- Theorem (2018 VMV & JMMM)

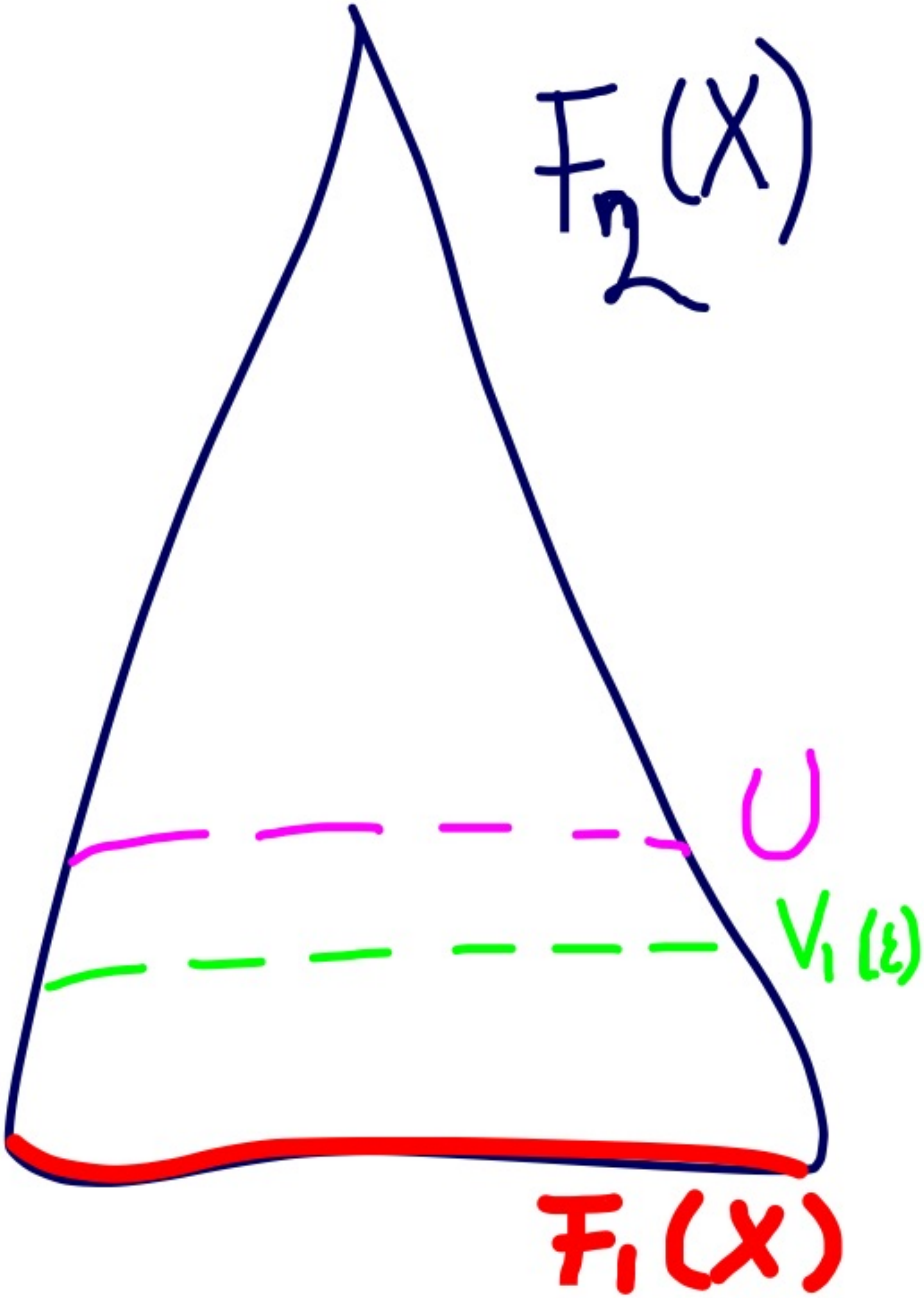
Let X be a nonlocally connected chainable continuum. Suppose that there is a monotone map $\pi : X \rightarrow [0,1]$ such that for each

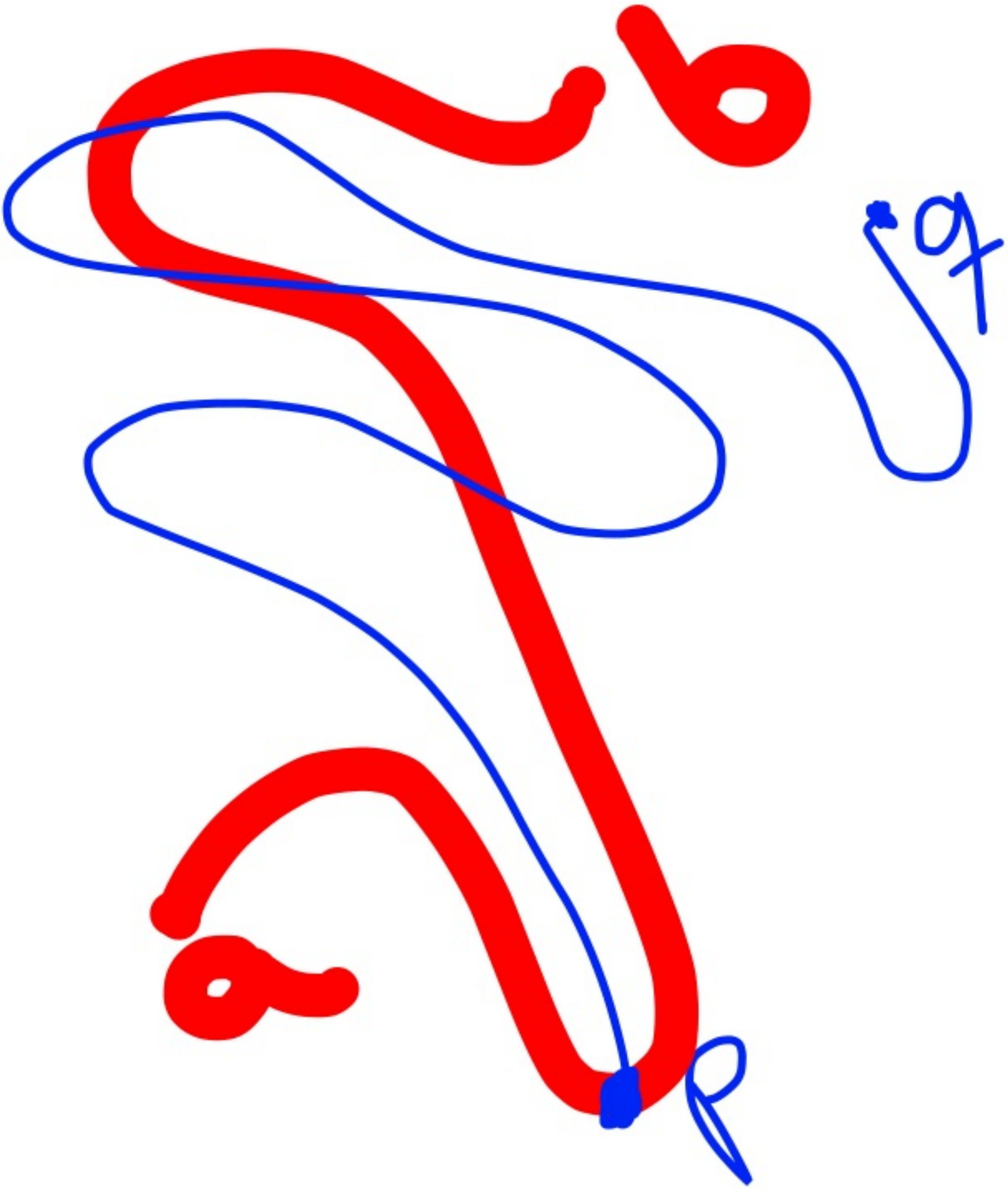
$a,b \in \pi^{-1}([a,b])$ with $a < b$. Then $F_1(X)$ is a weak cut continuum in $F_2(X)$.

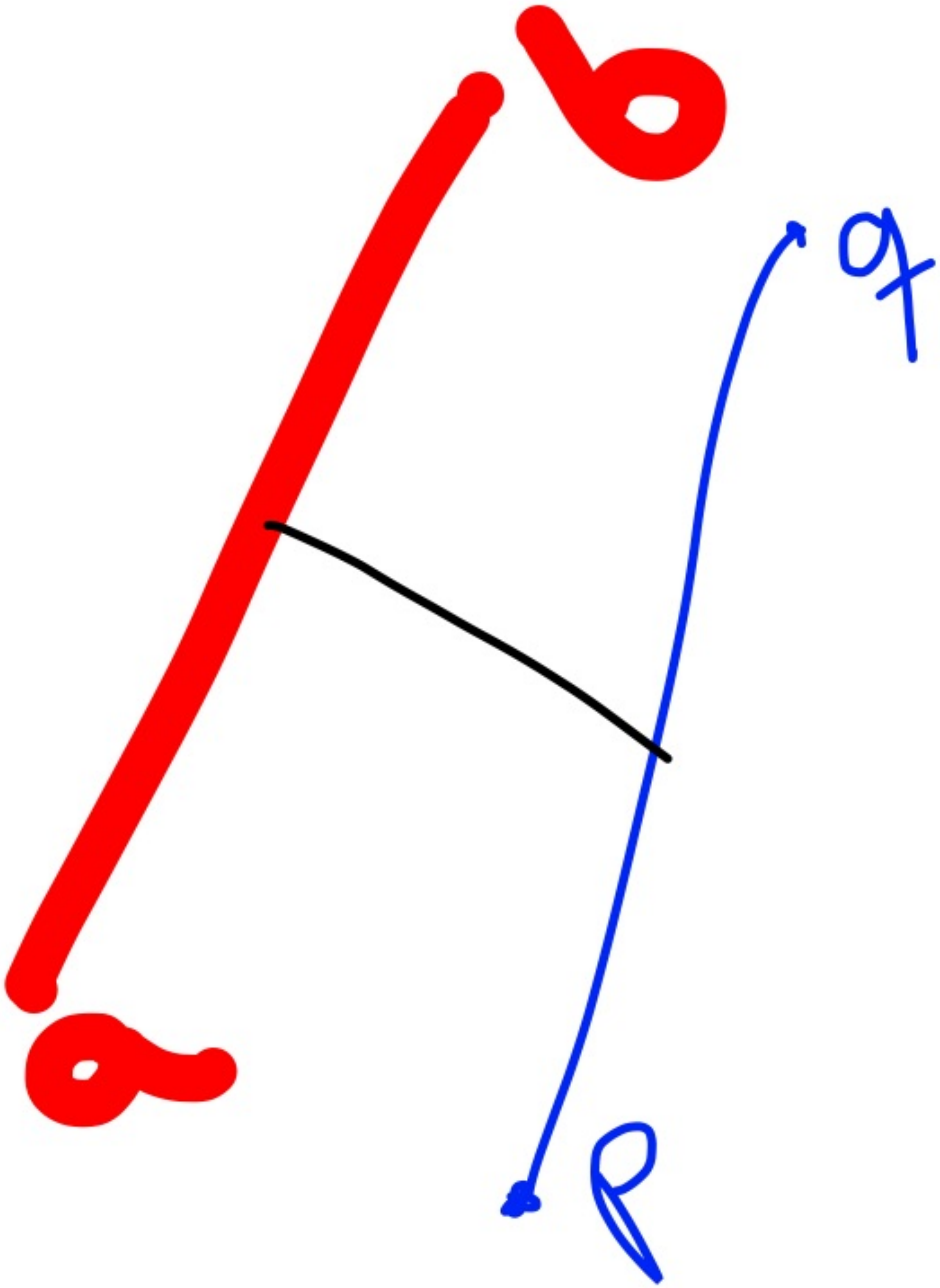
Theorem (VMV and JMM, 2017)

- For each arcwise connected continuum X and for n = 2, $F_1(X)$ is a non-cut set in $F_n(X)$



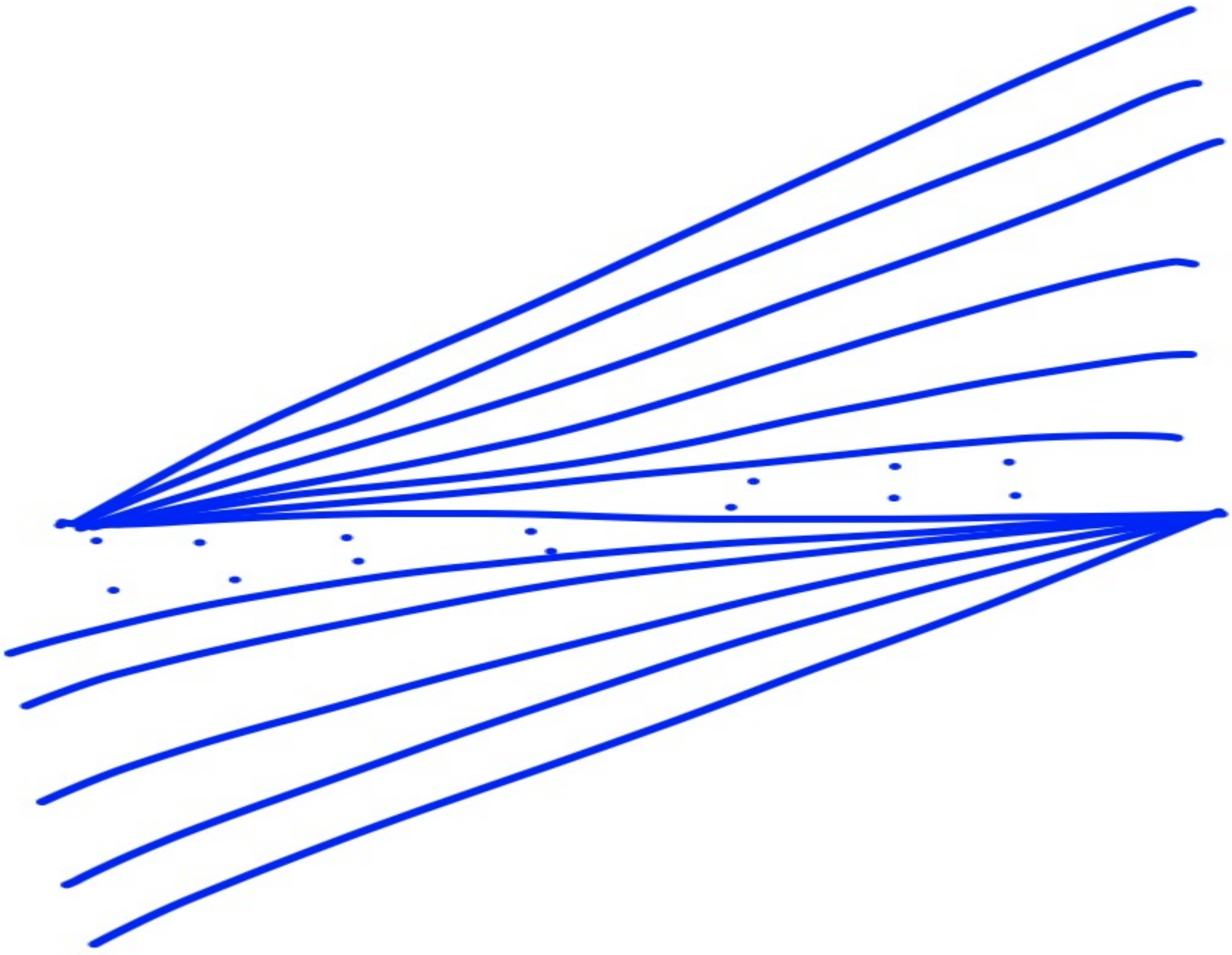

$F_2(X)$
U
$V_1(\varepsilon)$
$F_1(X)$

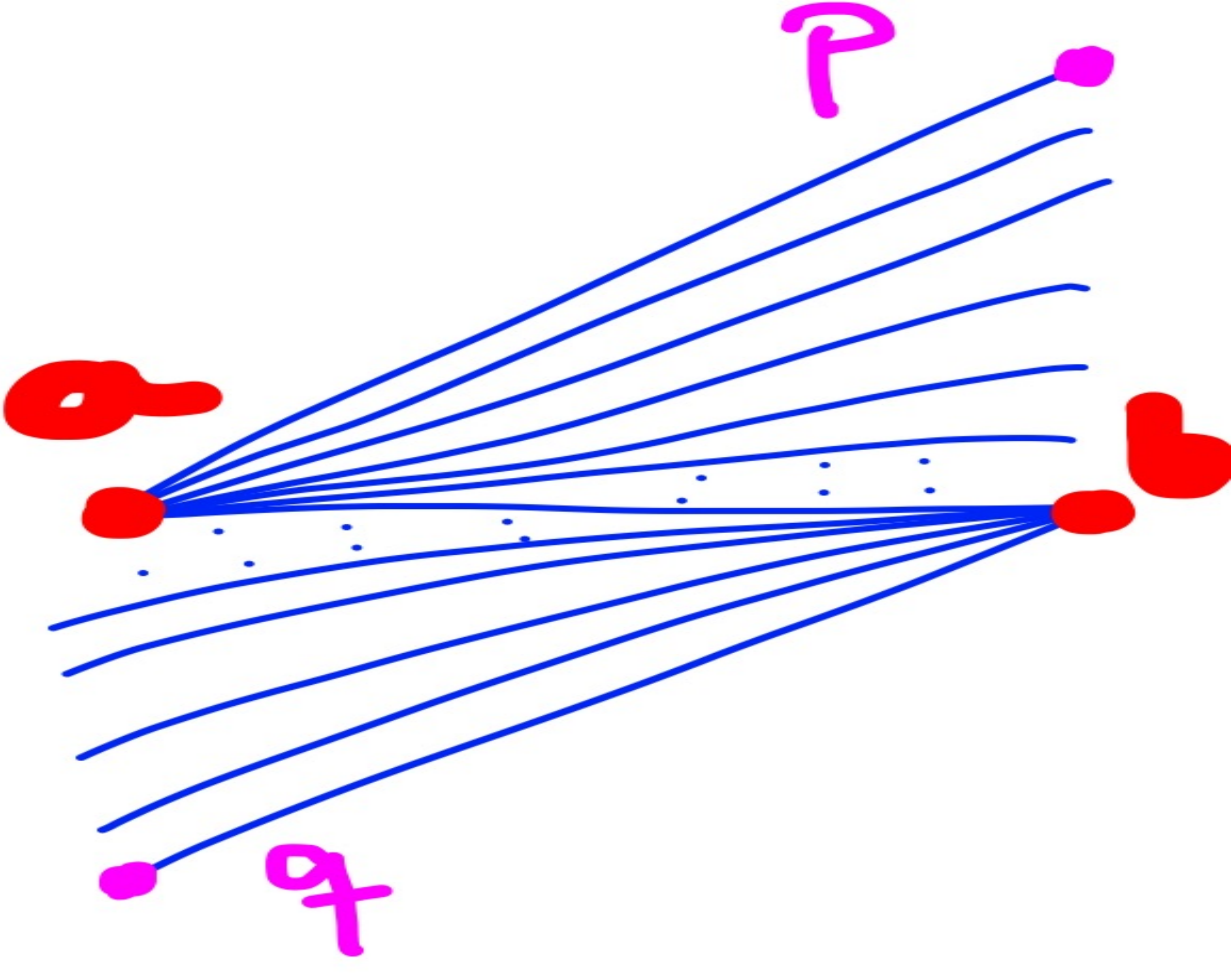

b
q
a
p

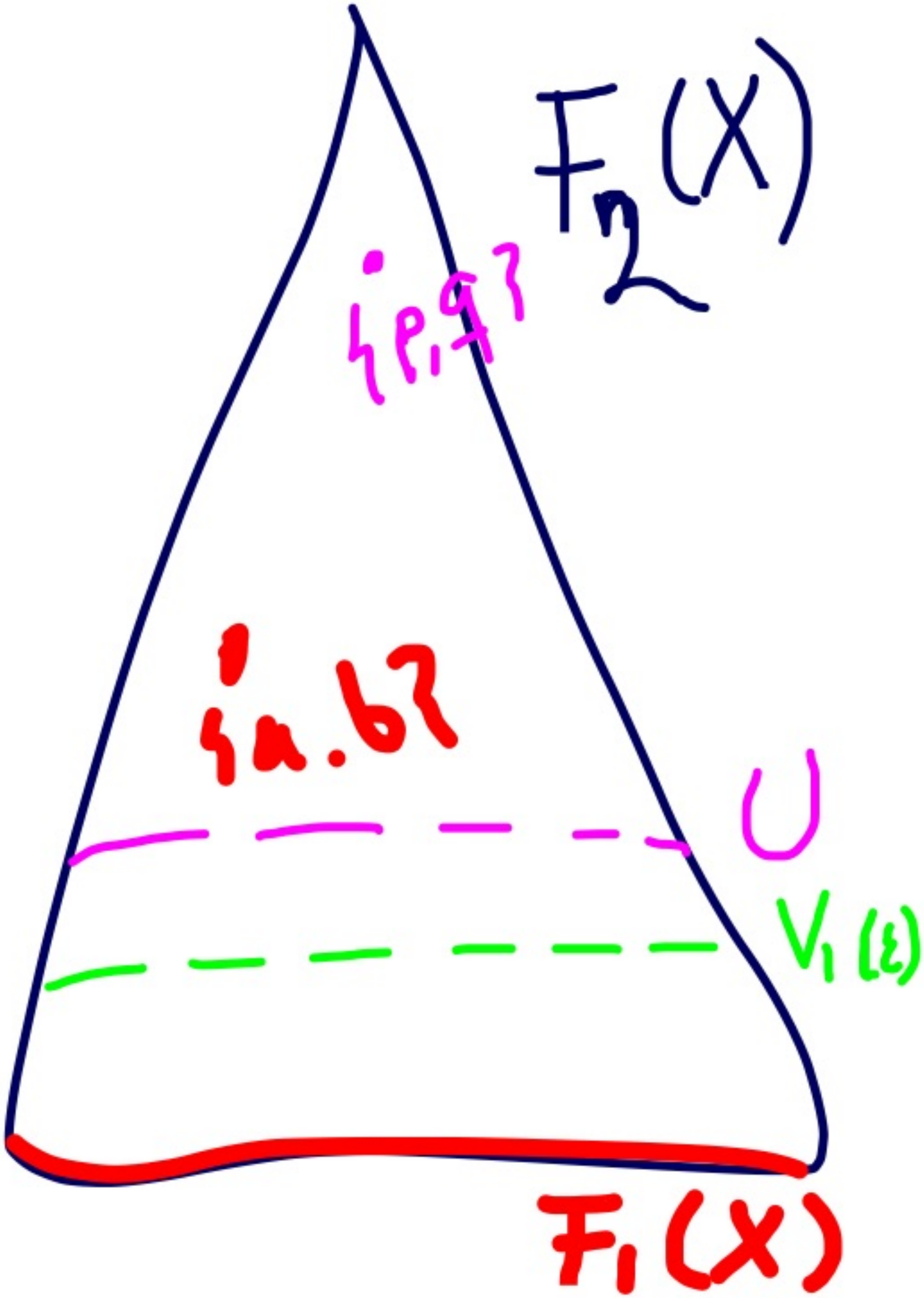

b
a
q
p

Theorem (VMV and JMM, 2017)

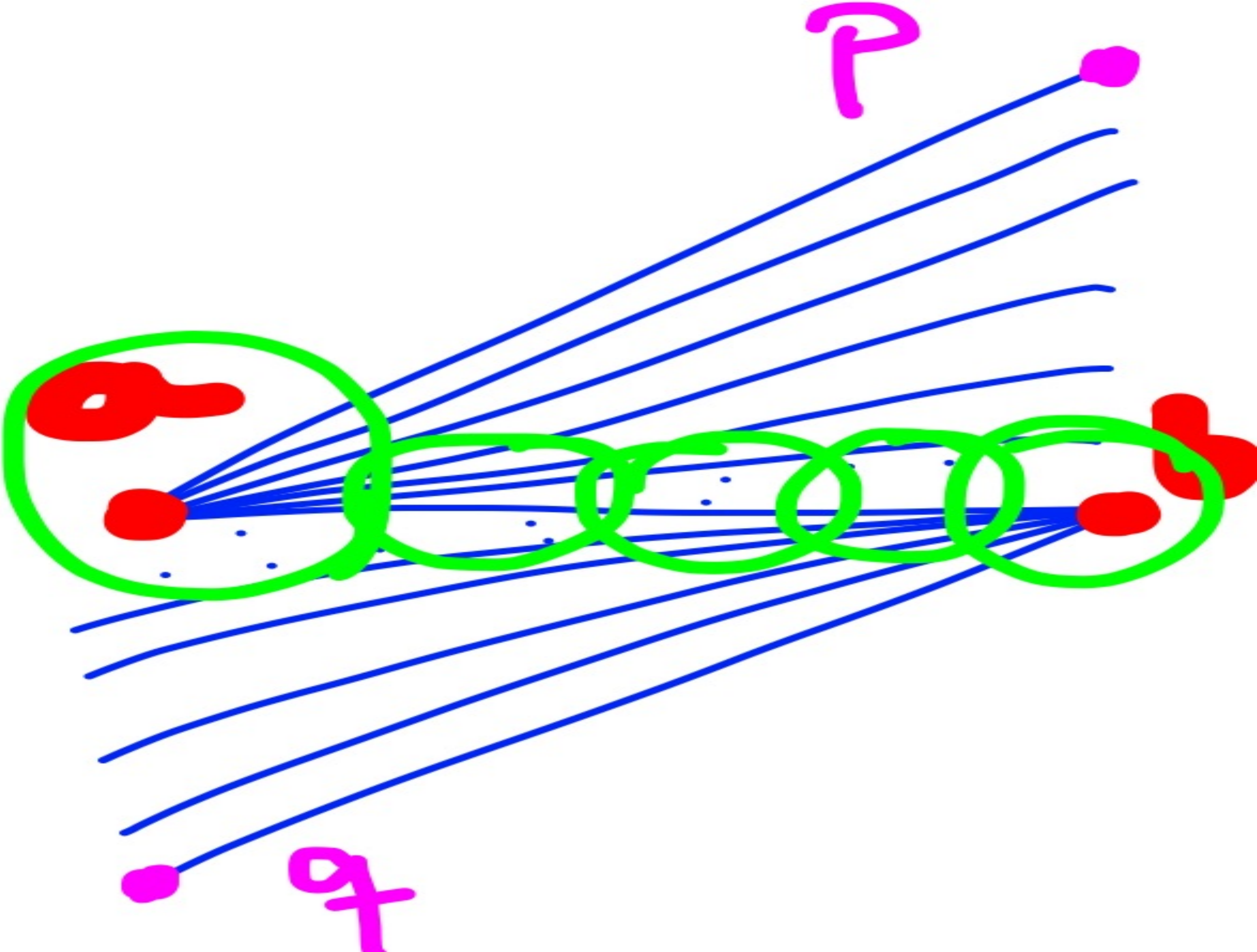
- There exists a dendroid X such that $F_1(X)$ is not a colocally connected set in $F_2(X)$.

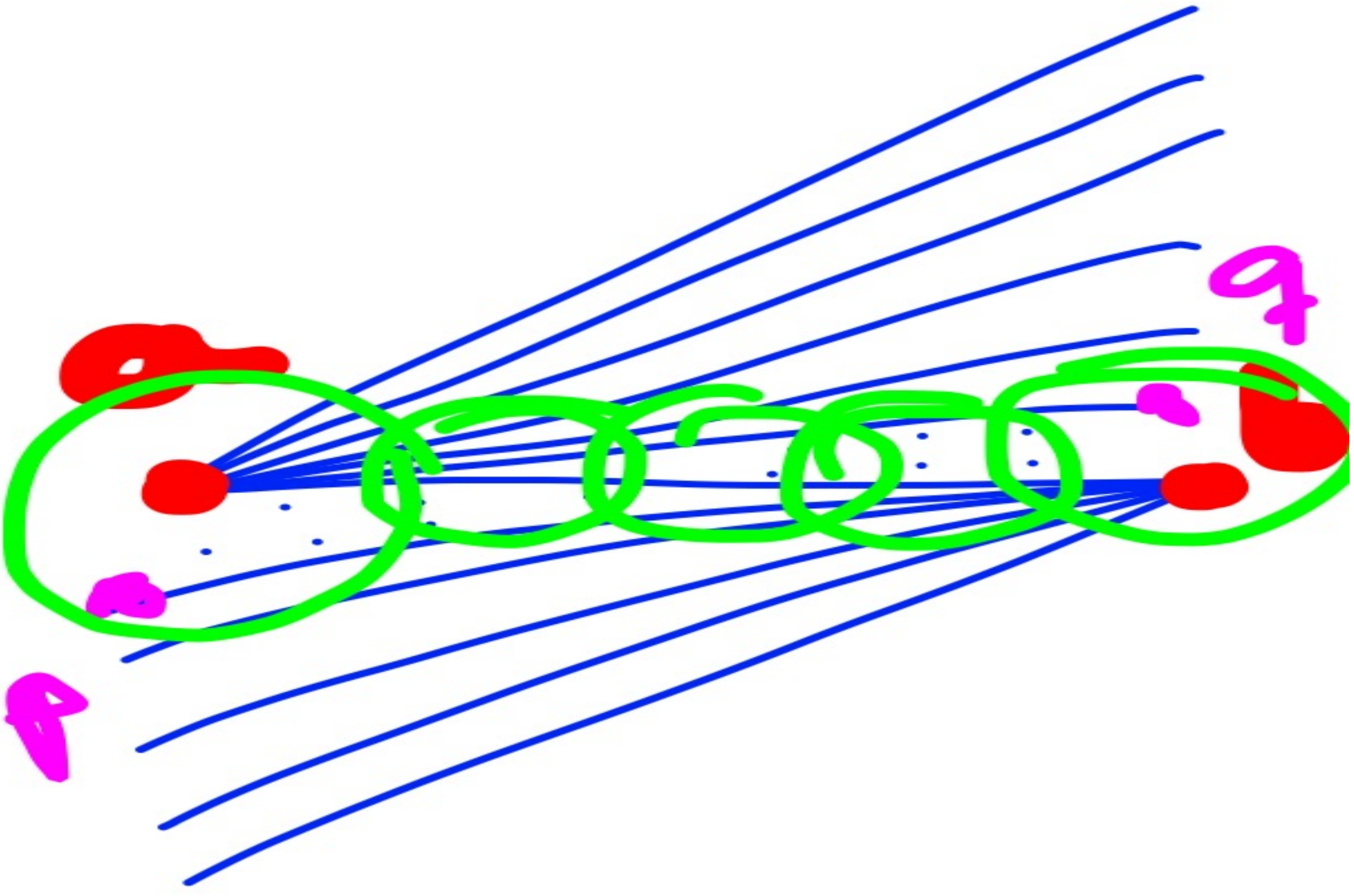



p
a
b
q



$F_2(X)$
$\{p,q\}$
$\{a,b\}$
U
$V_1(\varepsilon)$
$F_1(X)$


p
a
b
q


q
p

Colocally connected, Non-cut, Non-block and shore sets in Hyperspaces and Symmetric Products

- 1. For every continuum X and each integer n, $n \geq 2$, $F_1(X)$ is a colocal connected set in $C_n(X)$
- 2. For every locally connected continuum X and each positive integer n, $F_1(X)$ is a colocal connected set in $F_n(X)$

Colocally connected, Non-cut, Non-block and shore sets in Hyperspaces and Symmetric Products

- 3. For each continuum X and positive integer n, $F_1(X)$ is a non-block continuum in $F_n(X)$
- 4. The Knaster continuum K satisfies that $F_1(K)$ is not colocally connected in $F_2(X)$

Colocally connected, Non-cut, Non-block and shore sets in Hyperspaces and Symmetric Products

- 5. The sin(1/x) curve S satisfies that $F_1(s)$ is a weak cut continuum in $F_2(X)$

Colocally connected, Non-cut, Non-block and shore sets in Hyperspaces and Symmetric Products

- 8. For each arcwise connected continuum X, $F_1(X)$ is a non-cut set in Fn(X)
- 9. There exists a dendroid X such that $F_1(X)$ is not a colocally connected set in $F_2(X)$.

GRACIAS